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MEASUREMENT OF THERMAL CONDUCTIVITY OF A YELLOW BRASS AND CADMIUM AT LOW TEMPERATURES

A THESIS

Presented to

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by

Marvin Harris Cooper

In Partial Fulfillment

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MEASUREMENT OF THERMAL CONDUCTIVITY OF A YELLOW BRASS AND CADMIUM AT LOW TEMPERATURES

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Date approved by Chairman: 4/20/64

TABLE OF CONTENTS

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My sincere thanks go to Dr. W.T. Ziegler for his suggestion of this problem and for his untiring interest and assistance in its development and resolution. I also wish to thank the School of Chemical Engineering and the Engineering Experiment Station for facilities and materials used in this work. The schematic diagram of the thermal conductivity cryostat was included in this thesis with the kind permission of Mr. W.D. Bradbury, Jr.

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The thermal conductivity measurements described in this thesis were made for three purposes: (1) to obtain additional data for a free-machining yellow brass (62 per cent copper, 35 per cent zinc, and 3 per cent lead) and cadmium (99.95 per cent pure) at low temperatures, (2) to evaluate the operating behavior of the cryostat used here, and (3) to provide a basis for a comparison of this cryostat with one of a different design, both cryostats having been constructed in the Low Temperature Laboratory of the Engineering Experiment Station. Two previous independent investigations, made with these cryostats and reported in the theses of W. H. Wright and W. D. Bradbury, Jr., had shown significant differences in experimental results for the yellow brass. This investigation was carried out with the same equipment used by Bradbury, and all specimens involved in the three studies were from the same rod or batch.

In an experimental run, the specimen was suspended from a plate that, in turn, was attached to a heat sink in the form of a pot containing liquid nitrogen. The entire cryostat, evacuated to 10^{-5} mm. Hg or less, was immersed in liquid nitrogen. A heater, attached to the bottom end of the rod, introduced a known amount of heat into the system, and the e.m.f. readings of copper-constantan thermocouples, clamped to the rod at known points, indicated, through a calibration with a platinum resistance thermometer, the thermal gradient along the effective specimen length. Brass measurements were repeated in the temperature range

(84 to 118° K.) used by Bradbury and then the temperature range was extended to 146° K. Cadmium measurements were made at temperatures ranging from 64 to 119° K. Wright's data for both samples were obtained at temperatures from 80 to 275° K. During most of the experimental work, the two sets of difference thermocouples used by Wright were installed in the Bradbury cryostat, and their readings were recorded along with those of the Bradbury thermocouples. This inclusion of the Wright difference thermocouples permitted a comparison of the temperature scales used in the earlier work.

The thermal conductivity at a mean specimen temperature was calculated from the effective specimen length, effective heat input, specimen cross-sectional area, and thermal gradient. An important series of corrections for heat losses due to temperature drift, conduction, and radiation was applied to the apparent heat input.

The resulting thermal conductivities for the yellow brass and cadmium over the above-mentioned temperature ranges were in approximate agreement with the limited published data for similar specimens and conditions. Curves plotted from the brass and cadmium data obtained in this work also exhibited trends with temperature characteristic of alloys and pure metals, respectively.

The thermal conductivities for the yellow brass reported by Bradbury were consistently larger than those obtained by Wright and this work. This higher level of values and the marked upward trend at the higher temperatures can be attributed in part to the absence of corrections for conduction and radiation heat losses.

Over the range, 90 to 140° K., the yellow brass data from this work were about 2 per cent larger than those reported by Wright. From 80 to 120° K., the cadmium values obtained here with the Bradbury cryostat averaged 8 per cent larger than those reported by Wright.

Intercomparison of the sensitivity (i.e., dE/dT) of the difference thermocouples used by Wright with that of the single-junction thermocouples used in the Bradbury cryostat showed agreement to about 2.5 per cent over the range, 90 to 145° K. However, the sensitivities exhibited by the Wright difference thermocouples while in the Bradbury cryostat differed considerably from the calibration obtained by Wright; the Wright thermocouples used for the brass and cadmium measurements gave sensitivities about 3 and 11 per cent larger, respectively, in the Bradbury cryostat. Adjustment of the thermal conductivities reported by Wright for these higher thermocouple sensitivities resulted in values of the thermal conductivity that differed from those of this work, over the respective temperature ranges given above, by only 1 per cent for the yellow brass and 2.5 per cent for the cadmium. Thus, recalibration of the difference thermocouples used by Wright appears to have largely removed the discrepancy previously existing between the values obtained with the two cryostats.

CHAPTER I

INTRODUCTION

Evidence of the growing interest in the low-temperature field and the concomitant need for further knowledge of the properties of metals at these temperatures are to be found in the large number of recent publications on the subject. For example, Klemens(1), in the reference section of his exhaustive article*, listed forty-three papers reporting experimental work on the thermal conductivity of metals, twenty-nine of the papers having dates of 1950 or later. The measurement of metallic thermal conductivities is often of interest to both theorists and experimentalists; the former may compare the data with predicted values, and, of course, the latter would welcome any additional information which would aid them in the design of cryogenic equipment. While it is hoped that the data on a yellow brass and cadmium obtained by the work being reported here will prove useful to others, the primary purpose for making these measurements was a comparison of the behavior of two types of cryostats which had been constructed in the Low Temperature Laboratory of the Engineering Experiment Station. A discussion of this program, of which this investigation is a part, will be made later in this chapter.

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*Olsen and Rosenberg(2) have also reviewed the thermal conductivity of metals and alloys with regard to both theory and experiment. The compilations of experimental results made by Powell and Blanpied(3) and Johnson(4) provide convenient sources of data.

The limited scope of these introductory remarks makes the omission of many points and the inclusion of over-simplification inevitable. Therefore, the names and ideas given in the following brief outline of the meaning and measurement of low-temperature thermal conductivity in metals are to be considered illustrative rather than definitive.

The empirical connection between metallic electrical and thermal conductivities has been recognized for over a century*. The well-known Wiedemann-Franz-Lorenz law states that the thermal conductivity divided by the product of the electrical conductivity and the absolute temperature equals a constant. This relationship of the two conductivities led naturally to the "free" electron theory of metals initiated by Drude and Lorentz. This early electron theory, with its assumption of a Maxwell-Boltzmann distribution of the electron velocities, was to be affected, as was all atomic physics, by the advent of the quantum theory. Sommerfeld initiated this change by refining the Drude-Lorentz concept with the use of Fermi-Dirac statistics in place of the classical. Thus, with this approach, conduction of heat depends primarily upon the mean free path of the electrons. In dielectrics, on the other hand, heat is conducted only by the atomic lattice, the vibrations of which permit the transfer of thermal energy through the solid. Theoretical explanations of the lattice conductivity are based on an analogy to the kinetic theory of gases (the lattice waves or phonons) and the specific heat theory of Debye.

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*Much of the material in this section is based on the extensive theoretical discussion in the above-cited article by Klemens.

Experimental confirmations and denials of various elements of the theoretical proposals came from such prolific workers as Gruneisen and the group at Leiden, and, more recently, Mendelssohn and Rosenberg, at Oxford. The principal obstacle in early experimental work was the achieving of the low-temperature environment for the specimens to be tested. Steady progress in the securing of reliable data continued as improved facilities and methods evolved, and, since the end of World War II, cryogenic experiments have been greatly accelerated by the widespread installation of equipment capable of producing very low temperatures. Heat transport equations were formulated to predict the over-all thermal conductivities resulting from the complex interacting component mechanisms of heat conduction. Wilson, Makinson, and Sondheimer were leaders in this correlation of theoretical and experimental work. Generally, agreement between theory and actual measurement is good; however, there are still anomalies present and a careful inspection of the current state of the theory reveals the need for modifications(5).

The total thermal conductivity of a metal comprises two mutually interfering conductivities, the electronic and the lattice. Usually, with metals of high purity and a normal number of conduction electrons, the lattice mechanism contributes little to the conduction of heat compared to that afforded by the electrons. However, the lattice contribution becomes significant in alloys and must be considered in the estimation of total conductivity. The total thermal conductivity may be expressed as

$$K = K_e + K_g \qquad (1)$$

where

K = total or effective conductivity

K_e = conductivity due to electrons

K_g = conductivity due to the lattice

K_e and K_g are restricted by processes which scatter electrons and phonons, respectively. The scattering of these conducting "particles" produces, in effect, resistances to heat flow, and, therefore, discussions of the natures of K_e and K_g are conveniently made in terms of the inverse quantities, thermal resistivities. For the case of the electronic thermal resistivity,

$$1/K_e = W_e = W_i + W_o \tag{2}$$

where

W_e = electronic thermal resistivity

W_i = thermal resistivity due to thermal vibrations of the atoms in the lattice

W_o = thermal resistivity due to aperiodicity of the lattice structure

As mentioned above, the Wiedemann-Franz-Lorenz law relates W_o to the residual electrical resistance, ρ_o, (i.e., the electrical resistance at very low temperatures) by the expression:

$$W_o = \rho_o/L_o\, T \tag{3}$$

where

L_o = the Lorenz number = 2.45×10^{-8} watt-ohm/$^oK^2$.

It should be noted that the general statement of the law is usually valid, within a few per cent, at room temperature and above. At

temperatures of about one-fourth the Debye characteristic temperature, the ratio, $K\rho/T$, for pure metals becomes less, while those for alloys, semiconductors, and pure metals with low thermal conductivity become greater. The existence of appreciable lattice conduction accounts for the increase in the latter cases(6). Lattice conduction is lessened by deformations, such as those imposed by cold-working. This fact is the basis for the experimental method which determines the individual magnitudes of K_e and K_g. Thermal and electrical conductivities of a specimen are measured before and after cold-working and at temperatures low enough to avoid the scattering of electrons by lattice thermal vibrations. The K_e can be evaluated as equal to $L_o T/\rho_o$, and the decrease observed in the over-all thermal conductivity is taken as the value of K_g. Berman(7) reported that the electronic component amounted to 80 to 90 per cent of the measured thermal conductivity for a brass sample.

In turn, the lattice conductivity may be converted into a resistivity, W_g, and that differentiated into specific causative thermal resistivities(8):

$$\frac{1}{K_g} = W_g = W_B + W_E + W_I + W_U \qquad (4)$$

where

W_B = resistivity due to relative large lattice defects, including grain size and single crystal boundaries*

W_E = resistivity due to interaction of lattice with conduction electrons

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*Chubb(9) observed that thermal conductivity in cadmium may be as much as 20 per cent greater in single crystals than in polycrystalline metal.

W_I = resistivity due to lattice irregularities of atomic dimensions, i.e., small compared with the lattice wavelengths mainly effective in transferring heat, which are less than 10^{-5} cm. even at a few degrees, Kelvin.

W_U = resistivity due to the "Umklapp process", or interaction of phonons with each other (anharmonic coupling)

These thermal resistivities exert varying degrees of influence upon the total conductivity as the metal is cooled through a wide temperature range. A qualitative delineation by Makinson(8) of the roles played by these four resistances to lattice heat conduction is shown in Figure 1. The dotted line represents the lattice conductivity ($1/K_g = W_B + W_I + W_U$) for an insulator of high purity. The dashed line represents the contribution to the lattice conductivity arising from the interaction (W_E) of the electrons and the lattice waves. The notations adjacent to the solid curve, which represents the total lattice conductivity, K_g, indicate temperature regions in which these phonon-scattering processes are dominant.

A close description of the thermal conductivity of a pure metal at less than 40^o K. is given by the equation

$$1/K = AT^2 + B/T \qquad (5)$$

in which the two terms on the right are related to the lattice and impurities, respectively. The constants, A and B, are derived from experimental results. According to Rosenberg(5), a calculation of A depends on knowledge of the number of conducting electrons, the Debye characteristic temperature, and the thermal conductivity at high temperatures; B is equal to ρ_o/L_o.

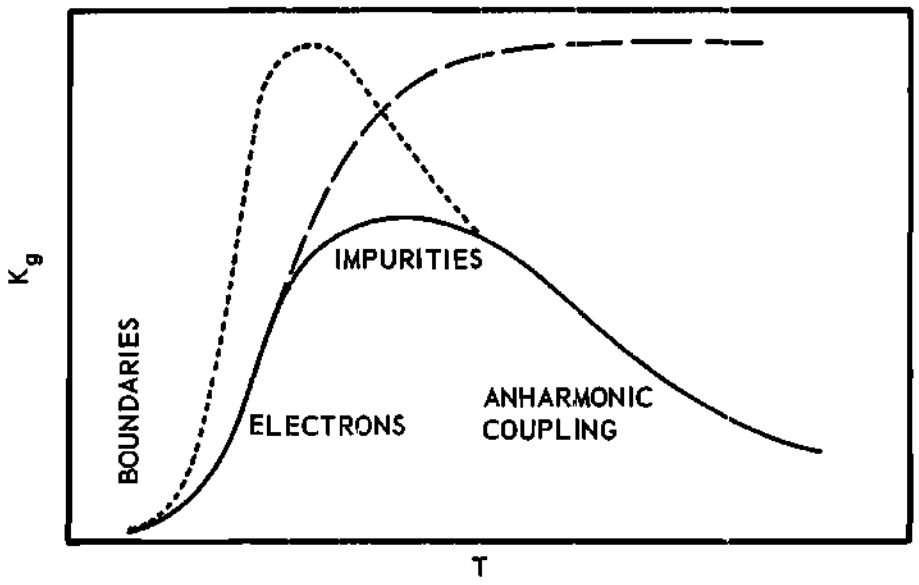

Figure 1. The Theoretical General Forms for the Lattice Thermal Conductivity. [After Makinson(8)].

In most cases, a pure* metal will exhibit increasing thermal conductivity with decreasing temperature and a linear approach to absolute zero after reaching a maximum. The electron-lattice scattering controls the shape of the curve to the right of the peak, and the impurity atoms and other lattice defects cause the decline to the left. The height of the maximum and its distance from the higher temperatures are increased by the removal of impurities from solid solution with the metal. Alloys, in general, display no maximum, but, instead, show a continuous decline in conductivity with decreasing temperature. Typical thermal conductivity curves for metals, alloys, and dielectrics in the low-temperature region are sketched in Figure 2.

The prior experimental work in this Low Temperature Laboratory on the free-machining yellow brass (approximate composition: 62 per cent copper, 35 per cent zinc, and 3 per cent lead) and cadmium (purity: 99.95 per cent) used in this work was that done by W. H. Wright(12) and, later, that by W. D. Bradbury, Jr.(13), who used a different cryostat, one of his design and construction and the one used for this work. The choice of the yellow brass as a specimen material stemmed from the importance of the alloy among the leaded brasses for use in apparatus construction; the relatively high purity of the cadmium sample made it an attractive subject for the examination of the thermal behavior of a metal at low temperatures. Wright measured the thermal conductivities

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*The sensitivity of the thermal conductivity to impurity caused Corruccini(10) to point out that "pure" metals might well be considered very dilute alloys. Furthermore, the results of the work by Powell, et al.(11) with very pure coppers indicated a presence of a term in the thermal resistivity causing a deviation from strict additivity of the lattice and impurity resistivities as stated in Equation 5 (Matthiessen's rule).

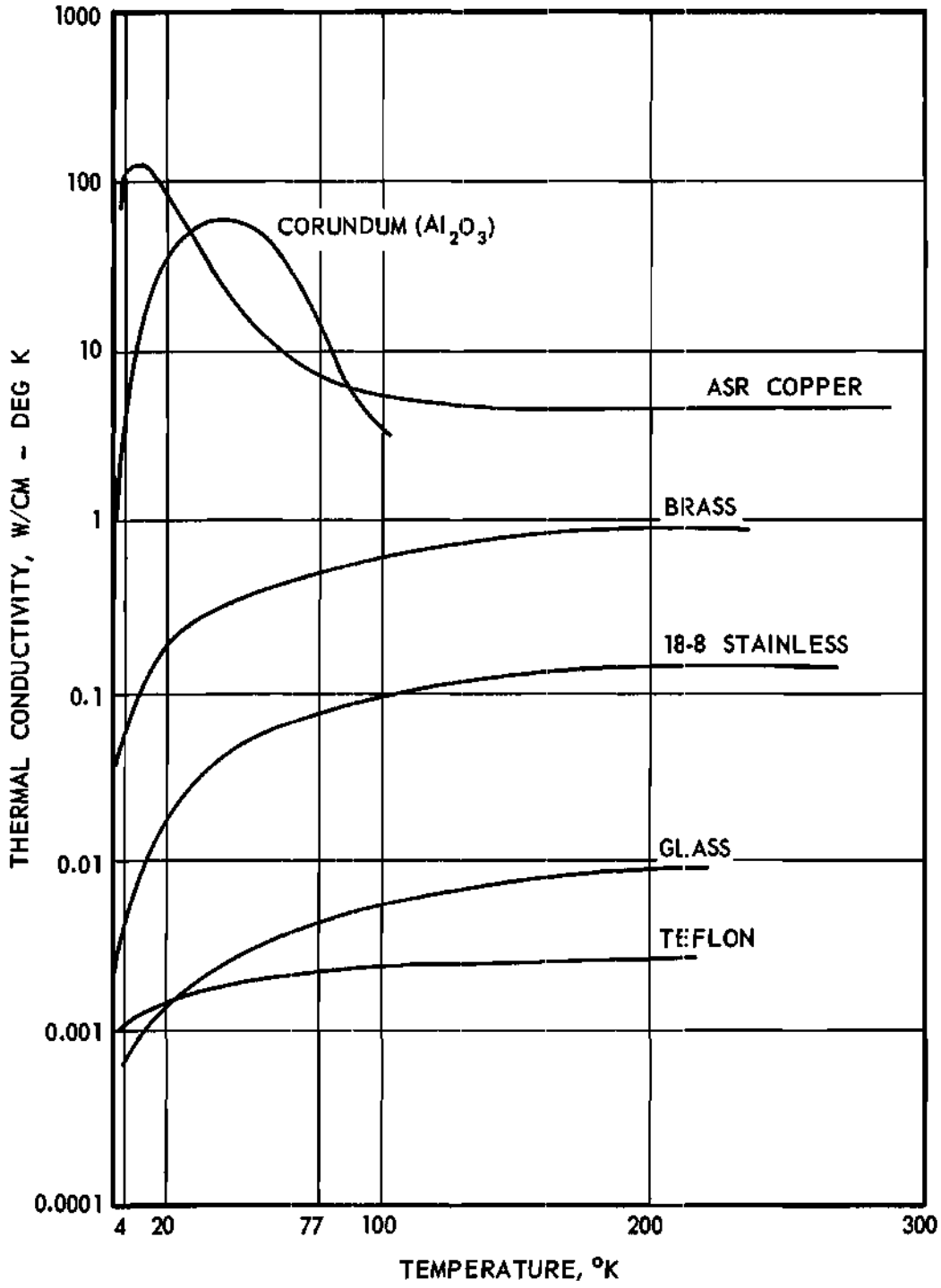

Figure 2. Characteristic Thermal Conductivity Curves for Various Materials. [After Corruccini(10)].

of the brass and the cadmium over the temperature range between 80 and 275° K.; Bradbury's measurements were of a preliminary nature, involving only the brass and temperatures from 84 to 118° K.

These limited measurements by Bradbury did not agree very well with those of Wright in the same temperature range, and this discrepancy suggested their possible use of different temperature scales. This matter was investigated here* by the insertion of the Wright difference thermocouples into the Bradbury cryostat and a repetition of the brass measurements. In this way, both sets of thermocouples could be compared under the same conditions. The measurements on cadmium in the Bradbury cryostat offered additional data which could be compared with that obtained by Wright. An extension of the temperature range up to 150° K. in this work provided an opportunity to observe more fully the operating characteristics of the Bradbury cryostat, particularly in regard to radiation losses and times required for attaining thermal equilibrium.

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*The experimental work for this investigation was completed in March, 1956.

CHAPTER II

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The equipment and facilities used in this work were the same as those used by Bradbury(13) for his measurements. A thorough description of the cryostat, specimen assembly, thermometric apparatus, electrical circuits, vacuum system, and other essential auxillary equipment can be found in his thesis. The following is a brief discussion of the laboratory work; detailed explanation of the experimental mechanics will be left to the treatment by Bradbury.

In Figure 3, the components of the cryostat are shown in schematic form. The specimen rod S was suspended from a metal plate E-E, which, in turn, was connected, structurally by rods N and thermally by copper wire F, to a liquid nitrogen pot P (heat sink). Attached to the specimen rod were two clamps, T1 and T2, which held thermocouples TC3 and TC4, respectively. A heater HT (heat source), attached to the lower end of the specimen rod, completed the suspended assembly. A secondary heater H1 was located on plate E-E. Placed concentrically outward of the specimen assembly were the shield R1 (lined with loosely supported aluminum foil) and the vacuum jacket J. Two additional thermocouples, pertinent to this work, were TC2, located at plate B-B (the top of the liquid nitrogen pot), and TD1, a difference thermocouple, connected between plate D-D (bottom of pot) and plate E-E. Shield R2, heater H2, and difference thermocouple TD2 were not used in this work, the shield having been removed.

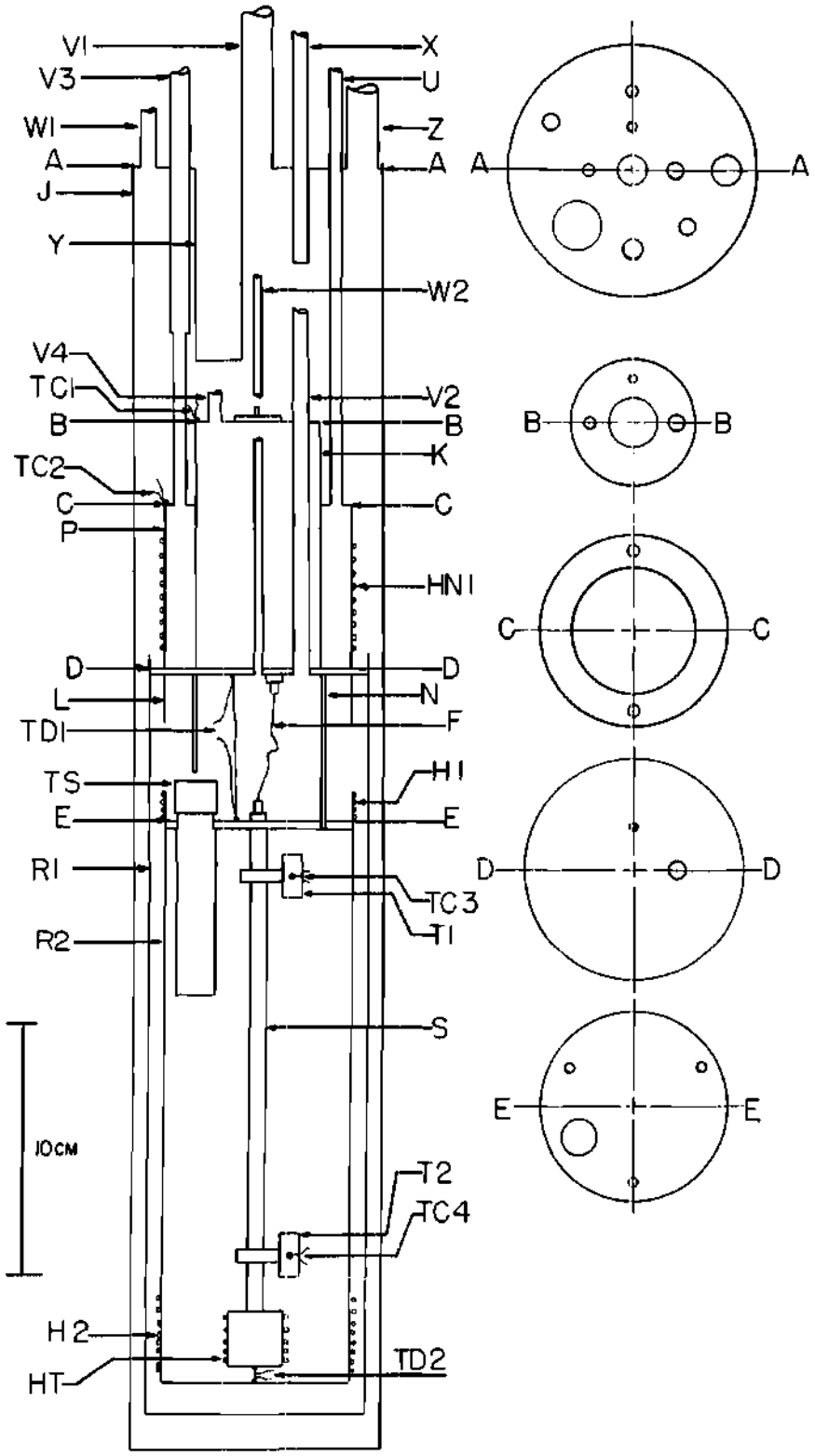

Figure 3. Schematic Diagram of Thermal Conductivity Cryostat.

The brass specimen rod studied here was the same as that used by Bradbury. It had been cut from the same rod of free-machining yellow brass (commercial stock, stated to be "half-hard" temper, from J. M. Tull, Atlanta) from which Wright(12) obtained his brass specimen. The composition of the brass was 62 per cent copper, 35 per cent zinc, and 3 per cent lead. The rod was turned down from 3/8-inch to 1/4-inch, the diameter of the specimen rod. For the cadmium measurements, a cast stick of the metal, 99.95 per cent pure and, again, from the same batch (A. D. Mackay, New York) as the cadmium specimen used by Wright, was machined down to a 1/4-inch specimen rod.

Copper-constantan thermocouples were used by both Wright and Bradbury. The temperature scale used by Bradbury depended upon a direct calibration of his thermocouples with a platinum resistance thermometer, which was calibrated by the National Bureau of Standards. (This reference thermometer served also as the basis for the temperature scale of Wright's measurements, except that his difference thermocouples were calibrated by an indirect procedure.) Bradbury's temperature scale was defined by the relationship,

$$E = 5879.7 - 5.3973\ T - 0.060976\ T^2 + 3.02 \times 10^{-6}\ T^3$$

where E is the e.m.f. in absolute microvolts and T is the temperature in degrees Kelvin. The reference junction was kept at 0°C., and the ice-point used was 273.16° K. These thermocouples and this temperature scale were used in the measurements discussed here.

Three minor modifications made to the Bradbury experimental set-up:

(1) the two sets of Wright difference thermocouples* were installed between the clamps T1 and T2 after the second brass run, (2) the heat leak wire F was removed between the ninth brass run and the first cadmium run, and (3) the rectified a.c. electrical source in the heater H1 circuit was supplanted by a d.c. source (batteries) after the ninth brass run.

In a typical experimental run, with the specimen assembly in place, the vacuum jacket was soldered completely to plate A-A, and the cryostat placed under vacuum with the combined operation of a mechanical pump, diffusion pump and cold trap (liquid nitrogen). When an acceptable vacuum (pressures of 10^{-5} mm. Hg or less) had been reached, the entire cryostat was immersed in liquid nitrogen contained in a large Dewar vessel. The liquid nitrogen pot P was then filled, the current through the heaters HT and H1 controlled at levels suitable for the desired temperature gradient along the specimen and temperature range, and the system allowed to approach thermal equilibrium, or, rather, near-steady-state conditions. Voltage measurements were made using a Leeds and Northrup Type K2 potentiometer (reproducibility: 0.3 μ v.); the energy supplied to the heater HT was measured with a conventional circuit containing a calibrated 1-ohm resistor. The times recorded for the potentiometer readings were either on the minute or half-minute, and a complete round of readings every twenty minutes was the usual rate of data collection during periods of thermal stability, i.e., with temperature drift rates of about 0.2° K. per hour. Attainment of thermally stable conditions was achieved more quickly with a minimum of heater adjustments and at lower

*In this work, these thermocouples were designated W1 and W2, corresponding to Wright's TCD1 and TCD2, which had been used for his cadmium and brass measurements.

temperatures, the latter causing less loss of liquid nitrogen in pot P, and, thereby, less change in the heat sink temperature. The reason for the removal of the copper wire F was the excessively high heat fluxes caused by its presence between plate E-E and the nitrogen pot. Several of the runs at the higher temperatures were completed in less than six hours in order to avoid adding more liquid nitrogen to the pot. On the other hand, in runs at lower temperatures, it was convenient to let the cryostat come to equilibrium overnight.

CHAPTER III

DISCUSSION OF RESULTS

Under steady-state conditions of axial heat flow, the thermal conductivity of the cylindrical specimen for sufficiently small temperature difference ΔT, can be stated as

$$K = \frac{L_e Q_e}{A \Delta T} \tag{6}$$

where

K = thermal conductivity, watts/cm.- ^{o}K.

L_e = effective specimen length, cm.

Q_e = effective heat input, watts

A = cross-sectional area of specimen, $cm.^2$

ΔT = temperature gradient along effective specimen length, ^{o}K.

Assuming that the dimensions of the specimen section of rod at room temperature were the same as those existing at the low temperatures, the measurement of thermal conductivities required only knowledge of Q_e and ΔT at times when the desired mean specimen temperature had been reached and satisfactorily maintained.

The determination of the effective specimen length, L_e, is discussed in Appendix B.

The e.m.f. readings from the copper-constantan thermocouples were converted to temperatures, according to the calibration made by

Bradbury. The difference between the temperatures indicated by TC3 and TC4 equaled the thermal gradient, ΔT, of Equation 6. The product of the voltage across the winding of heater HT and the current flowing through it was the apparent power (or heat) input to the heater and rod assembly. No correction was made for heat loss due to gas conduction; the magnitude of this loss is estimated in Appendix D. Corrections for heat losses due to temperature drift, conduction, and radiation resulted in the effective heat input, Q_e (see Appendix E). The calculation of thermal conductivity from experimental data is fully illustrated in Appendix F.

The thermal conductivity values gained from this investigation with the yellow brass and cadmium samples are shown in Table 1. In each instance, the temperature given is the arithmetic mean for the specimen. The temperature range for the yellow brass was 88 to 146° K., and the pairs of independent values at approximately 88, 92, 110 and 121° K. indicate the reproducibility of the measurements. The cadmium data involve temperatures from 64 to 119° K.

Before comparing the results for the brass and cadmium with those of Wright and Bradbury, a comparison with other published data on yellow brass and cadmium is helpful in establishing the general level of agreement with other published experimental data. The limited measurements of other investigators for several brasses of similar compositions are shown in Table 2, and these data, along with those of Table 1, are plotted in Figure 4. The leaded brass studied by Powell, et al.(14) is quite similar in composition to that studied in this research. The results of the present study are in approximate agreement

Table 1. Results: Thermal Conductivities for Yellow Brass and Cadmium

Temperature, ^{o}K.	K, watts/cm.-^{o}K.
Yellow Brass	
88.291	0.470
88.332	0.475
92.546	0.501
92.677	0.481
99.291	0.514
109.986	0.527
110.724	0.547
111.122	0.550
121.268	0.576
121.635	0.583
135.836	0.614
137.523	0.630
145.886	0.657
Cadmium	
64.780	1.124
67.987	1.077
74.128	1.067
87.979	1.033
99.980	0.982
109.028	0.990
118.938	0.965

Table 2. Published Thermal Conductivities for Yellow Brass

Worker and Sample Composition	Temperature °K	K watts/cm.-°K.
Lees(15)	103	0.732
30% Zn	113	0.758
	123	0.779
	148	0.837
Aoyama and Ito(16)	78	0.510
35.9% Zn		
Powell, et al.(14)	80	0.40[a]
35.7% Zn, 3.27% Pb, 1.0% Sn	100	0.46[a]
Kemp, et al.(17)	90	0.62[b]
31.87% Zn	90	0.52[c]

[a]Values read from a plotted curve

[b]Annealed

[c]Strained

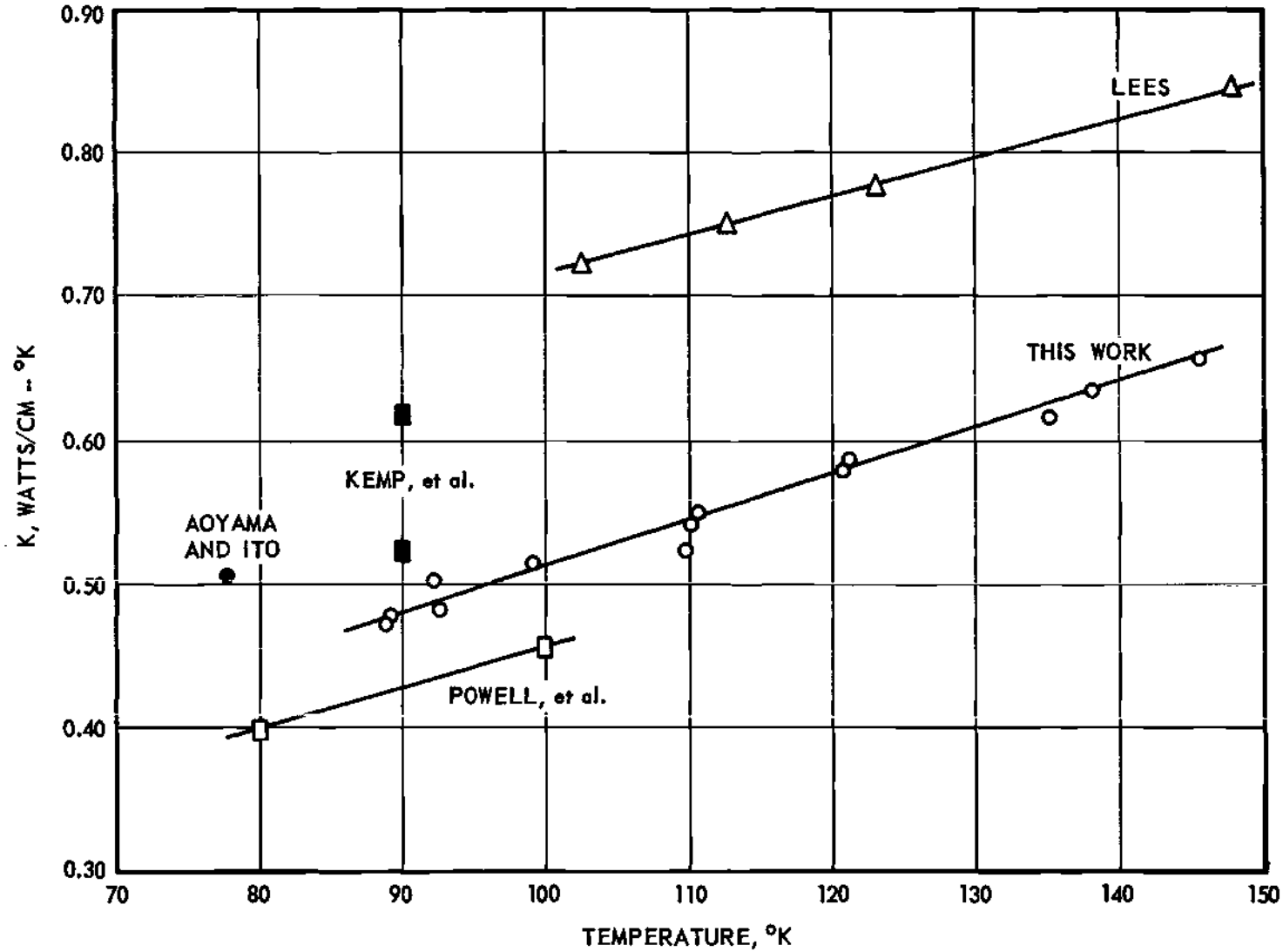

Figure 4. Comparison of Published Thermal Conductivities for Yellow Brass with Those of this Work.

with all except those reported by Lees(15). The exact strain condition of the brass studied in the present work is not known beyond the fact that it was stated to be of "half-hard" temper. The somewhat lower values of conductivity reported by Powell, et al. were for a material described as being of "hard" temper.

A similar graphical presentation of experimental results for cadmium from Tables 1 and 3 is given in Figure 5. In the region near 100°K., very good agreement is revealed by the curves of Lees(15) and this work: as little as one per cent difference is noted. The data offered by the other workers do not provide the same degree of confirmation. However, the cadmium data of Goens and Gruneisen(19) do bracket those of this work, the thermal conductivities from the samples with perpendicular(I) and parallel(II) crystal-rod axes being almost equidistant from the curve representing this work. Again, it may be pointed out that the data being reported here are typical in that the nearly pure cadmium metal exhibits the expected behavior of increasing thermal conductivity with decreasing temperature.

Comparison of the thermal conductivity measurements of Wright and Bradbury for yellow brass and cadmium with those made here has a more valid basis in that the specimens were of the same composition and history. Table 4 gives the thermal conductivity data reported by Wright and Bradbury in their theses. In Table 5, the Wright data appear again as values adjusted for differences in thermocouple sensitivity, a discussion of which is to be found in Appendix G. Figure 6 presents plots of the four sets of values for yellow brass; the adjusted data, identified as Wright II, are represented by the dashed curve. A similar comparison of the results for cadmium provided by Wright and this work is shown in Figure 7.

Table 3. Published Thermal Conductivities for Cadmium

Worker and Sample Description	Temperature °K.	K watts/cm.-°K
Lees(15)	103	1.005
"Pure redistilled";	113	1.000
lathe-turned from	123	0.996
cast stick	148	0.980
Eucken and Gehlhoff(18)	83	1.23
Kahlbaum; "chem. pure"		
Goens and Gruneisen(19)	81.8	0.92
Kahlbaum; "pure"; two	83.4	0.91
single crystals, each	91.6	0.91
with main crystal and rod axes parallel		
Kahlbaum; "pure";	82.4	1.13
single crystal with	82.8	1.12
main crystal and rod	91.4	1.12
axes perpendicular		

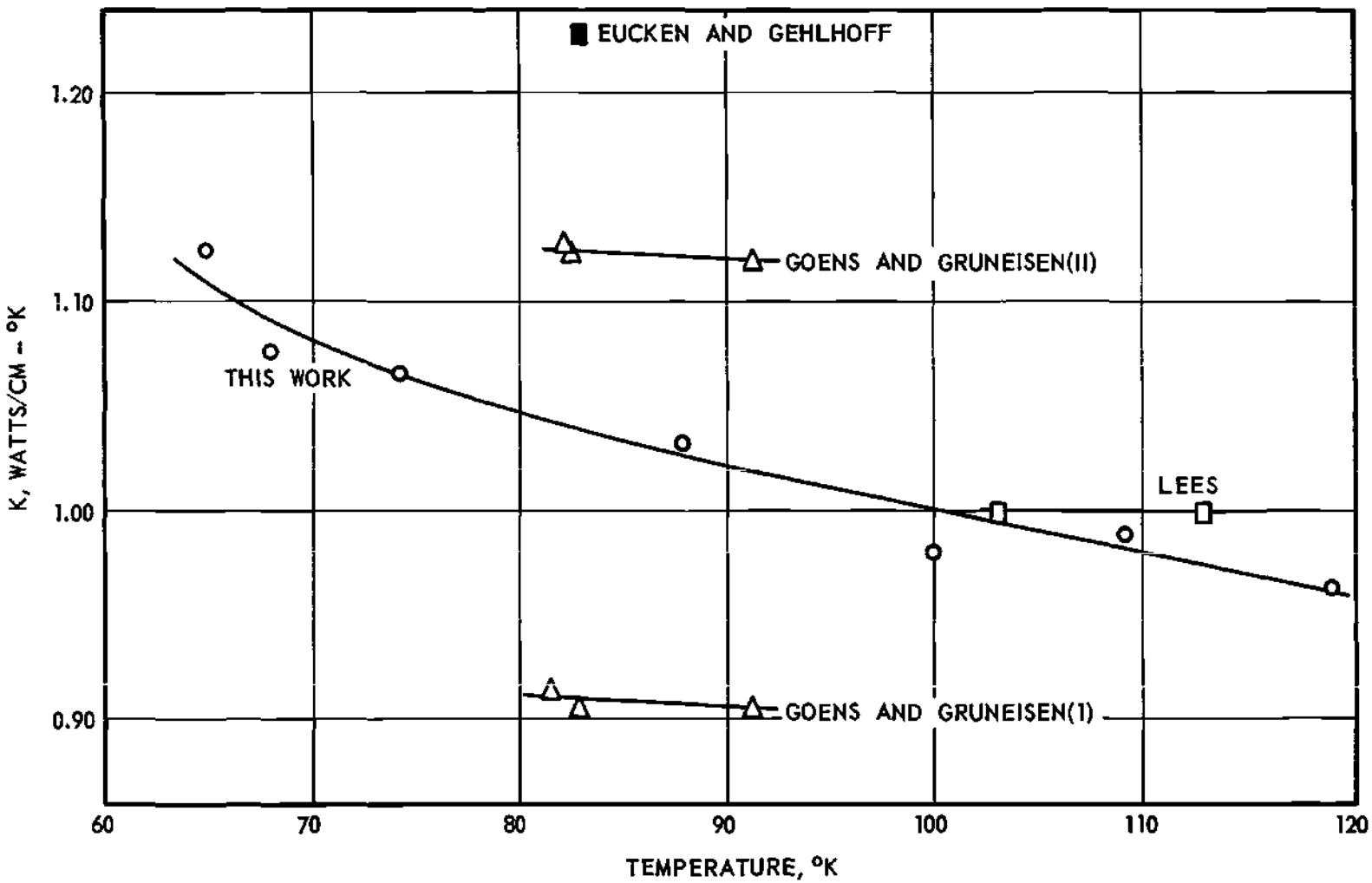

Figure 5. Comparison of Published Thermal Conductivities for Cadmium with Those of this Work.

Table 4. Wright and Bradbury Thermal Conductivity Data

Worker and Material	Temperature °K.	K watts/cm.-°K.
Wright(12)	79.88	0.434
Yellow brass	92.40	0.470
	113.36	0.542
	133.44	0.610
	158.78	0.708
Cadmium	81.51	0.940
	92.83	0.926
	113.92	0.914
	133.64	0.918
Bradbury(13)	84.67	0.486
Yellow brass	88.73	0.498
	98.90	0.528
	109.01	0.564
	118.24	0.605

Table 5. Wright Thermal Conductivity Data Adjusted for Variations in Thermocouple Sensitivity

Material	Temperature °K.	K' watts/cm.- K.
Yellow Brass	79.88	0.462
	92.40	0.491
	113.36	0.555
	133.44	0.618
	158.78	0.709
Cadmium	81.51	1.082
	92.83	1.041
	113.92	0.985
	133.64	0.961

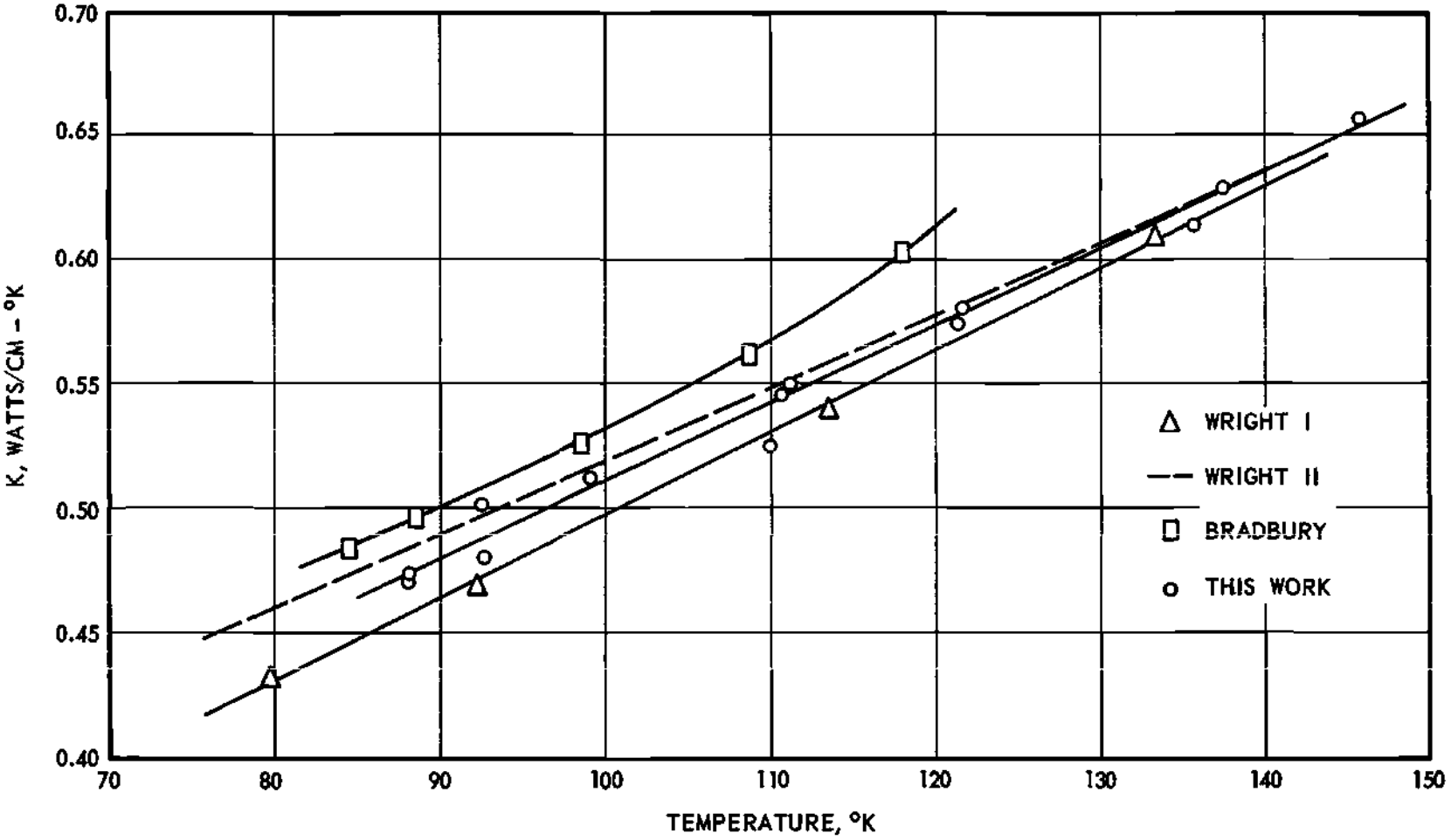

Figure 6. Comparison of Wright and Bradbury Thermal Conductivities for Yellow Brass with Those of this Work.

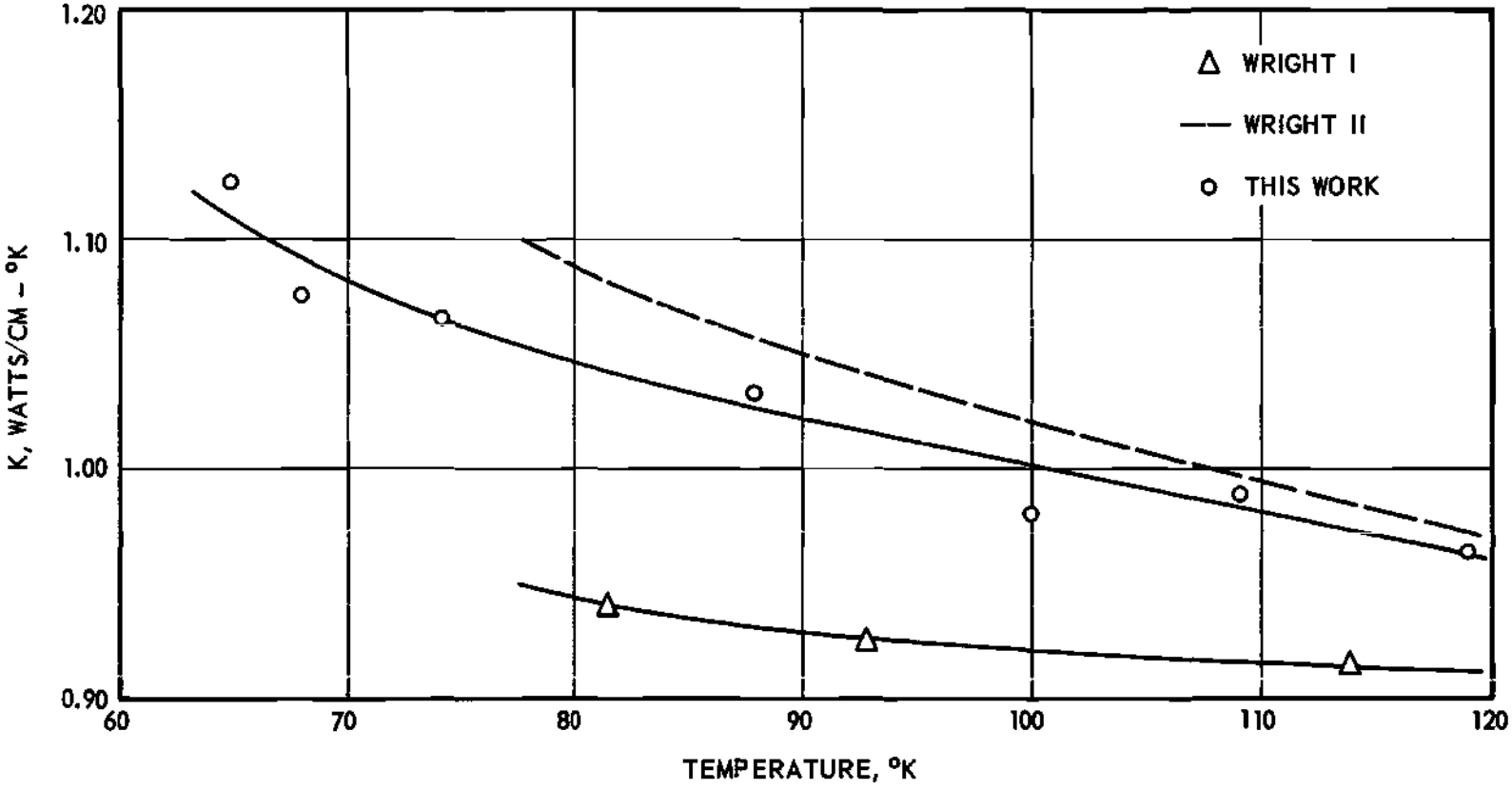

Figure 7. Comparison of Wright Thermal Conductivities for Cadmium with Those of this Work.

The Bradbury values for the yellow brass are consistently greater than the others, and the trend at the higher temperatures is one of an accelerated rise. This behavior of the Bradbury data is undoubtedly due to the absence of corrections for the conduction and radiation heat losses that become important at the higher temperatures.

As shown by Figures 6 and 7, the unadjusted thermal conductivities (Wright I) for both the yellow brass and cadmium are smaller than those reported here. For the brass, the data from this work are about 2 per cent greater than the Wright I data over the temperature range, 90 to 140° K. From 80 to 120° K., the cadmium values obtained here with the Bradbury cryostat average over 8 per cent greater than those of the Wright I curve.

From a comparison of temperature sensitivity (i.e. dE/dT) data for the Wright difference thermocouples, obtained with the Wright and Bradbury cryostats, it was found that the dE/dT data used by Wright was similarly less than that obtained through measurements made with the same Wright thermocouples, W1 and W2, in the Bradbury cryostat (see Figures 10 and 11, Appendix G). Since the lower sensitivity would result in larger calculated temperature gradients, the thermal conductivities reported by Wright would be correspondingly smaller than those calculated using the higher dE/dT observed for these difference thermocouples in the present work. After adjusting the reported Wright thermal conductivities for the differences in thermocouple sensitivity exhibited in the Wright and Bradbury cryostats, the agreement between the two sets of data is considerably improved. The adjusted values (Wright II), indicated by the dashed curves, are greater than those of

this work for both materials studied. Now, for the brass, over the temperature range, 90 to 140° K., the average difference drops to about 1 per cent, with intersection of the curves occurring near the higher temperature. Improvement is also readily seen in the case of cadmium: the data from this work are less than 2.5 per cent smaller than those of Wright II over the same temperature range (80 to 120° K.). In view of the relatively good agreement between the Wright thermocouple readings obtained in the Bradbury cryostat and the Bradbury thermocouple calibration (see Figure 9, Appendix G), it may therefore be reasoned that the Wright thermocouple calibration was in error, and that the differences in thermocouple sensitivities account for the major part of differences in the reported thermal conductivities.

CHAPTER IV

CONCLUSIONS AND RECOMMENDATIONS

The thermal conductivity of a free-machining yellow brass (62 per cent Cu, 35 per cent Zn, and 3 per cent Pb) has been measured over the range, 88.3 to 145.9° K. in a cryostat (I) designed by Bradbury(13). The values obtained agree to within about 2 per cent with values obtained by Wright(12) for a very nearly identical specimen, using a different experimental arrangement (Cryostat II). The disagreement between values obtained earlier by Bradbury, using Cryostat I, especially at the higher temperatures, is attributed to failure to correct for heat losses due to radiation from the specimen and conduction along thermocouple wires attached to the specimen. Similar measurements were made of the thermal conductivity of a cadmium specimen (99.95 per cent "pure") over the temperature range, 64 to 119° K. The values obtained were about 8 per cent larger than values reported by Wright, using Cryostat II.

Intercomparison of the sensitivity (i.e., dE/dT) of the difference thermocouples used by Wright in Cryostat II with that of the thermocouples used in Cryostat I showed agreement to about 2.5 per cent over the range, 90 to 145° K. However, the sensitivities exhibited by the Wright thermocouples while in Cryostat I differed considerably from the calibration obtained by Wright with Cryostat II; the Wright thermocouples used for the brass and cadmium measurements gave sensitivities about 3 and 11 per cent greater, respectively, in Cryostat I. Adjustment of the thermal

conductivities reported by Wright for these greater thermocouple sensitivities resulted in values of thermal conductivity that differed from those of this work, over the above-mentioned temperature ranges, by only 1 per cent for the yellow brass and 2.5 per cent for the cadmium.

Corrections for radiation losses sustained in the Bradbury cryostat cannot be expected to remain sufficiently accurate as the temperature level is extended upward very far, say above 110° K. Proper operation of the cryostat at relatively high temperatures will require the use of a compensation shield, with temperatures maintained along its length corresponding to those along the specimen at adjacent points. Consolidation and shielding of the thermocouple leads would also help reduce the radiation losses.

Of considerable practical importance to the operation of the Bradbury cryostat is the achievement of acceptable temperature drift rates within a reasonable length of time. If the applied temperature drift rate corrections are realistic, then much time would be saved by making measurements at higher drift rates. A liquid nitrogen pot of greater capacity would provide a more stable heat sink and, therefore, a more stable system.

Other improvements that are suggested by the experience gained so far with the cryostat are the supplanting of the thermocouple clamps with soldered connections to the specimens and the use of some other "heat leak" device rather than the present copper wire.

APPENDIX A

EXPERIMENTAL DATA

EXPERIMENTAL DATA

The three tables that follow contain the essential experimental data derived from this investigation.

Table 6: Bradbury thermocouple e.m.f. readings are listed under TC2, TC3, TC4 and TD1 (a difference couple). See Figure 4 for locations of these thermocouples. T3 and T4 are temperatures corresponding to TC3 and TC4 e.m.f. readings, respectively. ΔT is the temperature difference measured between TC3 and TC4; it is the thermal gradient along the specimen section of the rod.

Table 7: T_m is the mean specimen temperature. W1 and W2 are the Wright difference thermocouple e.m.f. readings. These are three-junction copper-constantan couples (Figure 8, Appendix B). I_h and E_h are heater HT current and voltage readings, respectively, and Q_o is their product or the apparent power (heat) input. In obtaining I_h, a correction was made for the small quantity of current that flows through a volt box that is in parallel with the heater HT; this correction is explained in Appendix F.

Table 8: The drift rates given here are those changes in e.m.f. readings with time observed for the Bradbury thermocouples at near-steady-state conditions during the experimental runs. The dE/dT values (changes in e.m.f. with temperature) were obtained by Bradbury in the calibration of the same copper-constantan thermocouples with a platinum resistance thermometer over the same temperature range.

Table 6. Experimental Data I

Run No.	TC2 μv.*	TC3 μv.	TC4 μv.	TD1 μv.	T3 °K	T4 °K	T °K
Brass:							
1	5090.9	4973.2	4886.0	89.2	85.582	91.000	5.148
2	5089.2	4901.8	4814.0	154.5	90.037	95.318	5.281
3	5080.3	4971.0	4888.6	81.8	85.722	90.842	5.120
4	5077.6	4900.4	4819.9	147.8	90.122	94.971	4.849
5	5073.3	4784.9	4705.9	253.6	97.023	101.558	4.535
6	5058.0	4594.0	4510.6	417.4	107.756	112.216	4.460
7	5046.1	4365.0	4287.9		119.717	123.552	3.835
8		4375.0	4292.5	643.5	119.212	123.325	4.113
9	5062.3	4069.3	3988.6	915.5	133.986	137.686	3.700
10	5081.4	4579.0	4498.0	462.3	108.568	112.879	4.311
11	5080.6	4571.6	4490.4	468.1	108.968	113.277	4.309
12	5069.4	4029.8	3954.3	979.3	135.807	139.239	3.432
13	5048.7	3839.8	3767.9	1145.7	144.325	147.448	3.123
Cadmium:							
1	5074.4	4951.1	4912.4	105.4	86.975	89.383	2.408
2	5063.3	4755.2	4712.0	280.4	98.746	101.215	2.469
3	5032.9	4592.8	4548.0	405.3	107.821	110.234	2.413
4	5051.8	4403.9	4356.8	602.6	117.745	120.130	2.385
5	5375.4	5289.4	5260.5	73.0	63.687	65.872	2.185
6	5426.3	5247.4	5216.2	159.6	66.844	69.130	2.286
7	5420.9	5162.3	5129.3	234.2	72.979	75.276	2.297

*μv = microvolts relative to a reference junction at 0° C.

Table 7. Experimental Data II

Run No.	T_m °K.	W1 μv.	W2 μv.	I_h amps.	E_h volts	Q_o watts
Brass:						
1	88.291			0.020680	3.2166	0.06652
2	92.677			0.020692	3.2212	0.06665
3	88.332	252.5	244.8	0.020426	3.1774	0.06490
4	92.546	246.8	239.7	0.020417	3.1780	0.06488
5	99.291	240.3	233.6	0.020385	3.1768	0.06476
6	109.986	252.8	247.2	0.020351	3.1768	0.06465
7	121.635	233.4	229.4	0.020324	3.1774	0.06458
8	121.268	245.9	241.8	0.020467	3.1996	0.06549
9	135.836	235.0	232.4	0.020447	3.2016	0.06546
10	110.724	242.6	238.8	0.020263	3.1632	0.06410
11	111.122	242.7	239.3	0.020262	3.1632	0.06409
12	137.523	220.3	221.7	0.020202	3.1638	0.06392
13	145.886	208.2	211.2	0.020133	3.1557	0.06353
Cadmium:						
1	87.979	124.2	117.1	0.020222	3.1554	0.06381
2	99.980	133.4	126.9	0.020099	3.1432	0.06318
3	109.028	136.6	131.4	0.020222	3.1668	0.06404
4	118.938	140.2	136.2	0.020051	3.1444	0.06305
5	64.780	97.4	89.4	0.020089	3.1186	0.06265
6	67.987	102.9	94.9	0.020033	3.1124	0.06235
7	74.128	106.7	98.9	0.019984	3.1094	0.06214

Table 8. Experimental Data III

Run No.	T_m °K.	Drift Rates, μv./hr. TC3	TC4	TD1	dE/dT (Bradbury) μv./° K.
Brass:					
1	88.291	3.0	4.5	3.9	
2	92.677	1.5	2.7	1.2	
3	88.332	6.0	6.0	2.7	16.1
4	92.546	4.0	6.0	8.0	16.9
5	99.291	10.0	12.0	9.0	17.4
6	109.986	4.5	4.5	2.5	18.7
7	121.635	9.6	9.3		20.1
8	121.268	0.6	0.9	2.0	20.1
9	135.836	0.3	0.5	0.3	21.8
10	110.724	2.1	1.8	1.3	18.8
11	111.122	0.6	0.9	1.1	18.9
12	137.523	3.9	3.6	2.4	22.0
13	145.886	6.0	5.1	3.0	23.0
Cadmium:					
1	87.979	0.7	0.5	0.2	16.0
2	99.980	0.1	0.1	0.7	17.5
3	109.028	2.7	2.7	0.1	18.6
4	118.938	2.4	2.4	0.3	19.7
5	64.780	3.0	2.4	1.8	13.2
6	67.987	1.2	0.9	1.2	13.6
7	74.128	1.5	1.5	1.2	14.4

APPENDIX B

THE EFFECTIVE LENGTH OF SPECIMEN

THE EFFECTIVE LENGTH OF SPECIMEN

Since the average width of the copper thermocouple clamps was 0.495 cm, the question of effective specimen length is certainly a valid one. While a rigorously precise determination of the effective length cannot be made, it seems reasonable that an examination of the two limiting cases can yield a close estimate. The symbols used in this discussion refer to Figure 8.

If the clamps were very good conductors and the rod a very poor conductor, the effective length would be that distance between the inner faces of the clamps, or L_1.

If the clamps were very poor conductors, the rod a very good conductor, and the gradient along the rod under each clamp the same in region L_1, the average temperature at each clamp would be located at its midpoint, and, therefore, the effective length would be L_2.

The case involved in this work was intermediate between the above two: the clamps and the rods were about equal in thermal conductivity. Consequently, the effective length chosen for evaluating thermal conductivity was one-half the sum of the lengths L_1 and L_2, or L_e. The effective lengths used was 12.535 cm. for the yellow brass specimen and 12.598 cm. for the cadmium specimen. The average cross-sectional areas of these specimens were 0.3167 $cm.^2$ and 0.3160 $cm.^2$, respectively. The total length of each specimen (from plate E-E to heater HT) was approximately 18 cm.

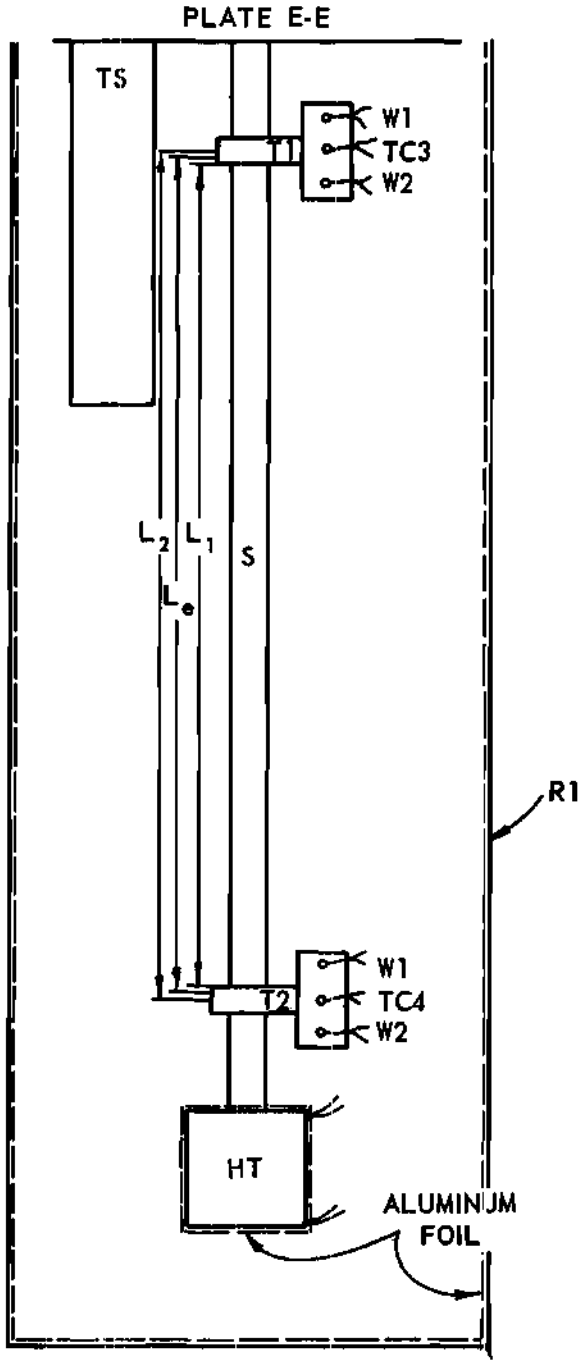

Figure 8. Specimen Assembly and Surroundings.

APPENDIX C

ESTIMATION OF ALUMINUM FOIL TEMPERATURE

ESTIMATION OF ALUMINUM FOIL TEMPERATURE

The heat loss due to radiation from the heater and specimen rod assembly to its surroundings is an important factor, particularly at the higher temperatures, and require corrections of the measured heat input before thermal conductivity calculations are begun. This and other heat input corrections are discussed in Appendix E. By covering the inside of the shield R1 loosely with aluminum foil, as shown in Figure 8, these radiation losses were reduced but not eliminated. Calculation of the radiation heat loss from the assembly depends upon an estimated value of the temperature of the enclosing aluminum foil; each experimental run (mean specimen temperature) produces a corresponding elevated aluminum foil temperature.

To simplify the estimation of the foil temperature, the system of heater and specimen assembly, inner aluminum foil, and copper shield R1 was assumed to be one of concentric spheres, and the aluminum foil was also assumed to be "floating", i.e., without significant direct contact with the copper shield. The plate E-E and the attached thermometer well TS were likewise considered a sphere within the quasi-spherical aluminum foil.

The radiation heat transfer from an inner (hotter) part to the outer (colder) part, such as from the aluminum foil (a) to the copper shield (c) is stated by McAdams(20) to be

$$q_{a \rightarrow c} = \frac{\sigma A_a E_a (T_c^4 - T_a^4)}{1 + \frac{A_a E_a (1-E_c)}{A_c E_c}} \qquad (7)$$

The A, E, and T terms represent the surface areas, emissivities, and temperatures of the two shields, respectively, and the Stefan-Boltzmann constant is shown with the conventional σ. Similar expressions can be set up for the radiation heat transfer from the plate E-E (p) to the aluminum foil and from the specimen assembly(s) to the foil. The emissivities selected for these several surfaces were approximations based on those reported by Cheung(21) for surfaces of a similar nature. Values for these emissivities and the areas of the surfaces are collected in Table 8. For the case of the heater and specimen assembly, the emissivity used was an effective one, viz., a value equal to the sum of emissivity-area products of each component of the assembly divided by the total area.

Assuming that

$$q_{a \rightleftarrows c} = q_{s \rightleftarrows a} + q_{p \rightleftarrows a} \tag{8}$$

the insertion of proper area and emissivity values gives, with consolidation,

$$28.42(T_c^4 - T_a^4) = 6.36(T_a^4 - T_s^4) + 13.59(T_a^4 - T_p^4) \tag{9}$$

and, solving for T_a^4,

$$T_a^4 = \frac{28.42T_c^4 + 6.36T_s^4 + 13.59T_p^4}{48.37} \tag{10}$$

T_c is taken to be constant at 78° K and T_p the temperature of the plate E-E and the thermometer well TS, was given a value five degrees less than that of the specimen assembly, which, in this case, was placed equal to

Table 9. Surface Areas and Emissivities

Component and Surface Description	Surface Area, $cm.^2$	Estimated Emissivity
Heater HT lacquered surface, wrapped with aluminum foil	33.0	0.043
Lower Section of Rod tarnished	6.1	0.11
Lower Copper Clamp T2 dull	5.8	0.2
Specimen Section of Rod tarnished	24.5	0.11
Total Rod and Heater Assembly	81.0	0.095
Copper Shield R1 dull	682.0	0.2
Aluminum Foil clean and bright	682.0	0.05
Plate E-E dull with some oxide	62.0	0.15
Copper Thermometer Well TS dull, grease-coated	30.0	0.25

T4, the temperature registered at the lower clamp. The values for T_a, obtained from the above relationship, form the basis for the radiation heat loss calculations described in Appendices E and F, the latter giving a complete set of sample calculations from experimental data.

APPENDIX D

HEAT LOSS DUE TO GAS CONDUCTION

HEAT LOSS DUE TO GAS CONDUCTION

According to Loeb(22), the following equation was proposed by Knudsen for the estimation of translational energy transfer between parallel plates that have absolutely rough surfaces (i.e., accommodation coefficients of unity) and are separated by a rarefied gas:

$$q_g = \left[\frac{2}{\pi}\right]^{\frac{1}{2}} \left[\frac{pG^{-\frac{1}{2}}(T_A - T_B)}{273\ T_g}\right] \tag{11}$$

where

q_g = energy transferred, ergs/cm.^{2}-sec.

p = pressure of gas, dynes/cm.^{2}

G = specific gravity of gas at 273° K. and a pressure of 1 dyne/cm.^{2}

T_A, T_B = temperature of plates, $^\circ$K.

T_g = temperature of gas, taken to be $(T_A + T_B)/2$

Assuming that this expression is applicable for an approximation of the heat loss through gas conduction from the heater and specimen assembly to the surrounding shield, and further assuming a mean specimen temperature, T_m, of 100° K., an aluminum foil temperature* of 78° K., and a gas (helium) pressure of 2×10^{-6} mm. Hg, Equation (11) becomes

$$q_g = (0.798)(2.666 \times 10^{-3}) \left[\frac{10^{-5}}{1.763}\right] \left[\frac{100 - 78}{273(89)}\right]$$

$$= 22.7 \text{ ergs/cm.}^2\text{-sec.}$$

*See Appendix C.

The heater and specimen rod assembly had a total surface area of 81 cm.^{2}, and, therefore,

$$q_g = 1.8 \times 10^{-4} \text{ watts}$$

This loss of heat represents less than 0.3 per cent of the average input of the heater HT used in the experiments, and, thus, has been neglected as a factor in the correction calculations. Other workers (6, 16, 23) have reported negligible gas conduction and convection heat losses at similar conditions of temperature and gas pressure.

APPENDIX E

HEAT INPUT CORRECTIONS

HEAT INPUT CORRECTIONS

For the ideal case of unidirectional heat flow along a rod, the basic equation is

$$\frac{dq}{dt} = -K\,A\,\frac{dT}{dL} \tag{12}$$

in which the terms are the same as those in Equation (6) of Chapter II, with the exception that, in this general case, the heat, q, flowing per unit time, t, is not necessarily a constant. If steady-state conditions are assumed, dq/dt becomes a constant, and, with a rod of constant cross-section, it can be shown that the thermal conductivity, K, will be an "average" K for the arithmetic average of the temperatures at the two points which define the effective length, L_e. On integration, one obtains

$$Q = \frac{K\,A\,\Delta T}{L_e} \tag{13}$$

or

$$K = \frac{L_e\,Q}{A\,\Delta T} \tag{14}$$

In actual practice, however, heat losses inherent to the experimental system cause the amount of heat that flows through the specimen section of the rod to be less than that measured at the heat source. In the experiments conducted in this work, there were several such factors present that necessitated corrections of the observed heat input, Q_o, found through measurements of power consumed by heater HT. Thus the Q of Equation (14) became a corrected or effective heat input, Q_e, which is the notation given in Equation (6).

The five corrections made to the measured heat input were due to temperature drift (q_1), conduction and radiation along the heater and lower clamp lead wires (q_2), radiation from the heater and the section of rod below clamp T2 (q_3), radiation from the specimen section of the rod (q_4), and conduction along the Wright thermocouple leads (q_5). The total application of these corrections provided the effective heat input,

$$Q_e = Q_o \left\{ 1 - \left[\frac{q_1}{Q_o} + \frac{q_2}{Q_o} + \frac{q_3}{Q_o} + \frac{q_4}{Q_o} + \frac{q_5}{Q_o} \right] \right\} \qquad (15)$$

which is then used in Equation (14) to determine the thermal conductivity of the specimen.

Derivation of General Equation for Correction of Heat Input

The following derivation of Equation (15) is similar to that given by Wright (in the Appendix of his thesis) for unidirectional flow of heat through a homogeneous, isotropic material under steady-state conditions and with constant cross-section and constant thermal conductivity over a small temperature range. It differs from the latter in that the entire specimen rod under study is not between the two measured temperatures, and, also, other heat losses, in addition to temperature drift, are considered in this case.

First to be taken up is the correction for temperature drift and radiation loss from specimen:

Consider a section of a circular rod between x and (x + dx). We will assume that

q_x = rate of heat flow across section at x

$q_{x + dx}$ = rate of heat flow across section at x + dx

then

$$q_x - q_{x+dx} = q \tag{16}$$

where q represents the accumulation of energy per unit time. Now,

$$q_x - q_{x+dx} = -KA\left[\left(\frac{dT}{dx}\right)_x - \left(\frac{dT}{dx}\right)_{x+dx}\right] = KA\frac{d^2T}{dx^2}\,dx \tag{17}$$

and

$$q = (C_p \rho A dx)\frac{dT}{dt} + q_r 2\pi r\,dx \tag{18}$$

where dT/dt = temperature rise per unit time ("drift rate")

$C_p \rho A\ dx$ = heat capacity of section between x and x + dx

q_r = radiation from surface per unit area of surface of section

Insertion of (17) and (18) in (16) gives

$$\frac{d^2T}{dx^2} = \frac{1}{\alpha}\left[a + \frac{q_r 2\pi r}{C_p \rho A}\right] = \frac{\beta}{\alpha} \tag{19}$$

where

$$\alpha = K/\rho C_p = \text{thermal diffusivity}$$

$$a = dT/dt$$

Assuming all quantities on the right-hand side of (19) are constant*, one can integrate (19) to obtain

$$\frac{dT}{dx} = \frac{\beta}{\alpha}x + N_1 \tag{20}$$

$$T = \frac{\beta}{\alpha}\frac{x^2}{2} + N_1 x + N_o \tag{21}$$

---

*For a somewhat more elaborate analysis of the problem in which q_r is assumed to be a linear function of the specimen temperature, see Reference (24).

Using the boundary conditions on the specimen section as

$$T = T_1 \quad , \quad x = x_1$$

$$T = T_2 \quad , \quad x = x_2$$

where $x_2 > x_1$ and $T_1 > T_2$, one obtains, on solving for N_1, the relation

$$N_1 = \left(\frac{T_2 - T_1}{x_2 - x_1}\right) - \frac{\beta}{2\alpha}\left(\frac{x_2^2 - x_1^2}{x_2 - x_1}\right) \tag{22}$$

and

$$\frac{dT}{dx} = \frac{\beta x}{\alpha} - \left[\left(\frac{T_1 - T_2}{x_2 - x_1}\right) + \frac{\beta}{2\alpha}(x_2 + x_1)\right] \tag{23}$$

Setting $x = x_1$, $dT/dx = (dT/dx)_{x_1}$

and substituting in the relation

$$q_{x_1} = -KA\left(\frac{dT}{dx}\right)_{x_1} \tag{24}$$

We obtain from (22), (23), and (24)

$$K = \left(\frac{x_2 - x_1}{T_1 - T_2}\right)\frac{1}{A}\left[q_{x_1} - \frac{\beta}{2} C_p \rho A(x_2 - x_1)\right] \tag{25}$$

Substitution for β gives

$$K = \left(\frac{x_2 - x_1}{T_1 - T_2}\right)\frac{1}{A}\left[q_{x_1} - \frac{C_p \rho A}{2}(x_2 - x_1)\left(a + \frac{q_r\, 2\pi r}{C_p \rho A}\right)\right]$$

or, on consolidating terms,

$$K = \left(\frac{x_2 - x_1}{T_1 \quad T_2}\right) \quad \frac{1}{A} \quad \left[q_{x_1} - \left(\frac{M_{x_2 - x_1} \; C_p}{2}\right) a - \frac{Q_r}{2}\right] \qquad (27)$$

where $M_{x_2 - x_1}$ = the mass of the specimen section $x_2 - x_1$

Q_r = $q_r 2\pi r(x_2 - x_1)$ = total radiation from specimen section $x_2 - x_1$.

Next, we turn to the calculation of energy entering the test section. This energy, q_{x_1}, may be related to Q_o, the energy supplied to the heater, by the relation

$$q_{x_1} = Q_o - q_1' - q_2 - q_3 - q_5 \qquad (28)$$

where q_1' = $a(M_h C_{ph} + M_{x_1 - x_o} C_{ps})$

q_2 = heat loss from wires to heater

q_3 = radiation loss from the heater and the specimen section $x_1 - x_o$

q_5 = heat loss along Wright thermocouples attached to lower clamp

Substitution of (28) in (27) results in the final expression for the calculation of K as

$$K = \left(\frac{x_2 - x_1}{T_1 - T_2}\right) \quad \frac{Q_e}{A} \qquad (29)$$

where $Q_e = Q_o \left[1 - \left(\frac{q_1}{Q_o} + \frac{q_2}{Q_o} + \frac{q_3}{Q_o} + \frac{q_4}{Q_o} + \frac{q_5}{Q_o}\right)\right]$ (15)

and $q_1 = a(M_h C_{ph} + M_{x_1 - x_o} C_{ps} + \frac{1}{2} M_{x_2 - x_1} C_{ps})$ (30)

$$Q_4 = Q_r/2$$

$$q_2, q_3, q_5 = \text{as defined above}$$

The corrections for each of these five heat losses will now be discussed in detail.

Correction for Temperature Drift

True steady-state conditions were not obtained with the cryostat because of the existence of thermal disturbances which, though slight, were able to produce changes in thermocouple readings. Any environmental incident, such as a change in heater output or liquid nitrogen level, can be a source of these disturbances. In most instances, the temperature change of the test specimen with time amounted to less than 0.2° K. per hour.

The temperature drift heat loss, q_1, was calculated with Equation (30). The drift rates being recorded as e.m.f. changes (dE/dt, microvolts per hour), it was convenient to use these values directly in Equation (30) with the following conversion:

$$a = dT/dt = \frac{dE/dt}{3600\ dE/dT} \qquad (31)$$

The factor of 3600 allows time to be expressed in seconds. Specific heat data for cadmium (25) were available, but none reported for brass; for this alloy, the Kopp-Neumann additive relation was used with data for copper (26), zinc (27), and lead (28).

Correction for Conduction Heat Losses along Wires from Heater and Lower Clamp

The seven B. & S. No. 34 copper wire leads that serve the heater and the lower clamp thermocouples form the major part of a bundle of wires that travels upward through the cryostat and out to the potentiometer system. Though the path taken is rather tortuous, with many loops about metal surfaces, the lowest temperature of these wires was assumed to be 78° K. and to occur at the well Y (Figure 3) to which the wires are cemented. The total wire length from this point to their lower ends is 170 cm. The temperature at the lower or hot ends was set equal to T4. The losses of heat from these wires result from conduction along the lengths of the wires and radiation from the surfaces.

A substitution of appropriate numerical values into Equation 13 established q_2, the heat lost by conduction. An estimation of the radiation losses, which were considered analogous to those occurring in the case of a thin fin connecting a heat source and sink (29), proved to be unsatisfactory. The magnitudes of such important factors as effective wire length, temperature, and emissivity and effective area of surfaces were essentially unknown, and preliminary calculations of the combined conduction and radiation losses by this method gave values that were excessively high. Therefore, while recognizing the existence of some radiation heat loss from the wires, it was felt that any radiation correction based on the information available would be too arbitrary to be useful and, consequently, has been neglected.

Correction for Radiation Heat Losses from Heater, Lower Rod, and Lower Clamp

If the radiation heat transfer from objects within the aluminum

foil shield of Figure 8 can be considered a case of radiation from a completely enclosed body to a comparatively large enclosing body, then the quantity of heat radiated, according to McAdams(20), is

$$q_3 = A_x E_x (T_x^4 - T_a^4) \qquad (32)$$

where A_x, E_x, and T_x are the surface area, emissivity, and temperature of the particular radiating object and T_a is the temperature of the aluminum foil. The temperature chosen for the enclosed parts was the same as T4; foil temperatures were provided by methods dealt with in Appendix C.

Correction for Radiation Heat Loss from Specimen Section of Rod

As in the above case of q_3, the radiation heat loss, q_4, from the specimen section was estimated with the use of appropriate area, emissivity, and temperature values in Equation (32). However, as stated in Equation (27), this heat loss is one-half the resulting calculated radiation heat loss, Q_r. The temperature of the specimen section was taken to be T_m, the mean specimen temperature.

Correction for Conduction Losses along the Wright Difference Thermocouple Wires

With the Wright thermocouples installed on the specimen assembly, some of the heat that reaches the lower clamp is bypassed around the specimen section of the rod by conduction along the six B. & S. No. 36 constantan wires and four B. & S. No. 38 copper wires that connect the upper and lower junctions of the two couples. All wires were five inches long, and the temperature gradient was T4 - T3. As with q_2, Equation (13) yielded the conduction losses, which were designated as q_5.

Calculated Values for Heat Input Corrections

Actual calculated values for the corrections discussed above are collected in Table 10, and a comparison can be made there of the magnitudes of the corrections at various temperature levels. The figures in Table 10 are fractions of observed heat input, permitting a ready inspection of the relative importance of the test imperfections. It may be noted that, below 100° K., the total correction does not exceed four per cent for cadmium, and, except for one run with an unusually high temperature drift rate, the brass correction is below six per cent in that temperature range.

Table 10. Heat Input Corrections

T_m °K.	Run No.	$\frac{q_1}{Q_o}$	$\frac{q_2}{Q_o}$	$\frac{q_3}{Q_o}$	$\frac{q_4}{Q_o}$	$\frac{q_5}{Q_o}$
Brass:						
88.291	1	0.0161	0.0074	0.0068	0.0013	
88.332	3	0.0264	0.0074	0.0068	0.0014	0.0108
92.546	4	0.0223	0.0099	0.0090	0.0028	0.0103
92.677	2	0.0156	0.0098	0.0091	0.0025	
99.291	5	0.0495	0.0138	0.0134	0.0047	0.0096
109.986	6	0.0204	0.0200	0.0224	0.0080	0.0095
110.724	10	0.0092	0.0206	0.0234	0.0084	0.0092
111.122	11	0.0036	0.0208	0.0236	0.0085	0.0092
121.268	8	0.0035	0.0261	0.0346	0.0124	0.0086
121.635	7	0.0424	0.0266	0.0354	0.0126	0.0082
135.836	9	0.0019	0.0344	0.0576	0.0217	0.0078
137.523	12	0.0168	0.0362	0.0618	0.0235	0.0074
145.886	13	0.0241	0.0413	0.0822	0.0298	0.0068
Cadmium:						
64.780	5	0.0111				0.0048
67.987	6	0.0041				0.0050
74.128	7	0.0064				0.0051
87.979	1	0.0026	0.0067	0.0063	0.0016	0.0052
99.980	2	0.0004	0.0138	0.0136	0.0049	0.0054
109.028	3	0.0117	0.0190	0.0207	0.0079	0.0052
118.938	4	0.0104	0.0251	0.0318	0.0120	0.0052

APPENDIX F

SAMPLE CALCULATIONS

SAMPLE CALCULATIONS

The following calculations are based on the data obtained from the fourth run (see Tables 6,7,8) made with the brass specimen. They should serve to illustrate the derivation of thermal conductivities from all runs.

As previously mentioned, the thermal conductivity of a material can be defined by the relationship,

$$K = \frac{L_e Q_e}{A \Delta T} \tag{6}$$

Providing accurate numerical counterparts for the terms on the right constitutes the experimental problem. The length and cross-sectional area of the specimen and the thermal gradient are readily identified; the effective heat input, however, is a far more recondite quantity, requiring the major part of the calculations in the form of apparent heat input corrections.

The effective specimen length, as defined in Appendix B, was 12.535 cm. and the cross-sectional area averaged 0.3167 $cm.^2$ In Table 6, the e.m.f. readings of TC3 and TC4 at near-steady-state conditions were recorded as 4900.4 and 4819.9 microvolts, respectively. The Bradbury thermocouple calibration shows that the TC3 reading indicates a temperature between 90 and 91° K., i.e.,

	4902.4	μv.	90° K.
TC3	4900.4		
	4886.0		91

The temperature, T3, at the clamp T1 is

$$90 + 2.0/16.4 = 90.122^{\circ} \text{ K.}$$

In the same way, a temperature of 94.971° K. is found for T4 at clamp T2. The gradient, Δ T, along the specimen is difference between these two temperatures:

$$\Delta T = T4 - T3 = 94.971 - 90.122 = 4.849^{\circ} \text{ K.}$$

and the mean temperature is

$$T_m = \frac{94.971 + 90.122}{2} = 92.546^{\circ} \text{ K.}$$

For the calculation of the apparent heat input to the specimen heater, values for the current and voltage maintained at the heater are necessary. The electrical circuit for heater HT (see Chapter III of Bradbury's thesis) contained a standard resistor in series with the heater winding, and the potential drop measured across this known resistance permitted the calculation of the total current flowing through the heater circuit. With a standard resistor, R_s, of 1.00044 absolute ohms and a potential drop, E_s, of 0.020447 volts, the total current was

$$I_s = \frac{E_s}{R_s} = \frac{0.020447}{1.00044}$$

$$= 0.020438 \text{ ampere}$$

However, since the measurement of the potential drop across the heater winding required a volt box along with the potentiometer, the very

small portion of the current that flowed through this 150,000-ohm resistor (in parallel with the heater) must also be determined. Readings were made with the volt box in the ratio of 1 to 200; in order to obtain the actual heater winding potential, E_h, the reading, E_{vb}, had to be multiplied by 200:

$$E_h = 200\ E_{vb} = 200(0.015890)$$
$$= 3.1780 \text{ volts}$$

The volt box current would therefore be

$$I_{vb} = \frac{200\ E_{vb}}{R_{vb}} = \frac{3.1780}{150,000}$$
$$= 0.000021 \text{ ampere}$$

Subtracting this volt box current from the total heater circuit current calculated above, the amount of current that flowed through the heater was

$$I_h = I_s - I_{vb}$$
$$= 0.020447 - 0.000021$$
$$= 0.020417 \text{ ampere}$$

Finally, the power (or heat) input to the heater is expressed as

$$Q_o = I_h\ E_h$$
$$= (0.020417)\ (3.1780)$$
$$= 0.06488 \text{ watt}$$

The corrections to be applied to this apparent heat input for the several heat losses are explained in detail in Appendix E. The first heat loss to be considered here is that resulting from temperature drift. Table 8 provides drifts rates of 4.0μv./hr. for TC3 and 6.00μv./hr. for TC4. Using the average of the two readings and converting to °K./sec., the drift rate becomes

$$5.0/(3600)(16.9) = 0.0000822^{\circ}\ \text{K./sec.}$$

The specific heats of the copper and the brass at the temperature T4 are 0.241 and 0.252 joules/gm.-°K., respectively. The weights of the specimen assembly components below the mid-point of the lower clamp are

copper	heater	37.0 gms.
	clamp	12.5
brass	sample	30.2
	lower rod	7.5

Substituting in Equation 30,

$$q_1 = \left[(37.0 + 12.5)(0.241) + (7.5 + 30.2/2)(0.252)\right] 8.22 \times 10^{-5}$$

$$= 0.001448 \text{ watt}$$

Expressed as a fraction of the apparent heat input,

$$\frac{q_1}{Q_o} = \frac{0.001448}{0.06488} = 0.0223$$

The seven copper wire leads attached to the heater and the lower clamp were 170 cm. long and 0.0001836 cm.^{2} in cross-sectional area

(diameter: 0.006 in.). The thermal conductivity (3) of the copper wire was taken to be 5.0 watts/cm.-^{o}K. in the temperature range encountered. With a gradient of 17.0^{o} K.(T4-78), the conduction loss, according to Equation 13 is

$$q_2 = \frac{(5.0)(0.0001836)(17.0)}{170}$$

$$= 0.000643 \text{ watt}$$

and

$$\frac{q_2}{Q_o} = \frac{0.000643}{0.06488} = 0.0099$$

For the conditions of this run, the temperature of the aluminum foil is computed from Equation 10 to be

$$T_a^4 = \frac{28.42(78)^4 + 6.36(95.0)^4 + 13.59(90.0)^4}{48.37}$$

$$T_a = 84.0^{o} \text{ K.}$$

With this value of T_a, the radiation heat losses from the heater, lower rod, and lower clamp--which collectively have an effective emissivity of 0.0722 and a surface area of 44.9 cm.^{2} (Table 9)--are determined with Equation 32:

$$q_3 = (5.71 \times 10^{-12})(44.9)(0.0722)\left[(95.0)^4 - (84.0)^4 \right]$$

$$= 0.000583 \text{ watt}$$

and

$$\frac{q_3}{Q_o} = \frac{0.000583}{0.06488} = 0.0090$$

The specimen section of the rod possessed an emissivity of 0.11 and a surface area of 24.5 cm.^{2}. An estimate of the heat loss from this section by radiation is obtained with Equations (27) and (32):

$$q_4 = (0.5)(5.71 \times 10^{-12})(24.5)(0.11)\left[(92.5)^4 - (84.0)^4\right]$$

$$= 0.000181 \text{ watt}$$

and

$$\frac{q_4}{Q_o} = \frac{0.000181}{0.6488} = 0.0028$$

Finally, calculation of the heat losses by conduction along the Wright thermocouple wires brings Equation 13 into use:

(copper) $$q_5 = \frac{(5.0)(0.00007967)(4)(4.849)}{12.7}$$

$$= 0.000609 \text{ watt}$$

(constantan)* $$q_5 = \frac{(0.2)(0.0001265)(6)(4.849)}{12.7}$$

$$= 0.0000579 \text{ watt}$$

The total heat loss from the wires is, therefore,

$$q_5 = 0.000667 \text{ watt}$$

and

$$\frac{q_5}{Q_o} = \frac{0.000667}{0.06488} = 0.0103$$

---

*The thermal conductivity of the constantan was assumed to be 0.2 watt/cm.-^{o}K. in this temperature range (3).

The resulting effective heat input is now, after the enumeration of the experimental heat losses,

$$Q_e = Q_o \left\{ 1 - \left[\frac{q_1}{Q_o} + \frac{q_2}{Q_o} + \frac{q_3}{Q_o} + \frac{q_4}{Q_o} + \frac{q_5}{Q_o} \right] \right\} \qquad (15)$$

$$= 0.06488(1 - 0.0543)$$

$$= 0.06488(0.9457) = 0.06136 \text{ watt}$$

Returning to Equation 6, the thermal conductivity of the specimen section of the brass rod at a mean temperature of 92.546° K. is computed to be (see Table 1)

$$K = \frac{(12.535)(0.06136)}{(0.3167)(4.849)}$$

$$= 0.501 \text{ watt/cm.-}^{\circ}\text{K.}$$

APPENDIX G

COMPARISON OF WRIGHT AND BRADBURY THERMOCOUPLES

COMPARISON OF THE WRIGHT AND BRADBURY THERMOCOUPLES

The thermal conductivity data obtained with the Wright and Bradbury experimental equipment and methods depend upon the calibration of the thermocouples used in the respective cryostats. The inclusion of the Wright difference thermocouples in the Bradbury cryostat during most of the runs made in this investigation afforded an opportunity to compare the temperature sensitivity (in microvolts/degree) under identical conditions.

Each Wright difference thermocouple being actually a series of three copper-constantan thermocouples, the resulting e.m.f. readings, such as those appearing in Table 7 or Appendix A, must be divided by three and by the temperature gradient, ΔT, so that the comparison with the Bradbury data can be made on a per-junction and per-degree basis. These reduced readings, along with the Bradbury calibration figures for the single copper-constantan thermocouples (Table 8), are compiled in Table 10 adjacent to the pertinent mean specimen temperatures. Figure 9 also facilitates the comparison of the simultaneous measurements with its plots of dE/dT versus temperature.

Agreement between W1 (Wright's TCD1, used by him for cadmium) and the Bradbury calibration is best at 117° K., while that between W2 (Wright's TCD2, used by him for brass) and the Bradbury data occurs at 91° K. In the region from 100 to 130° K., the three sets of thermocouples provide very similar values; here W1 and W2 differ from the Bradbury calibration by a maximum of 2.5 per cent. Indeed, in the range, 90 to 145° K., the average deviation between Wright thermocouples and the Bradbury calibration is less than 2.5 per cent. Below 90° K., greater

Table 11. Variation of dE/dT with Temperature for Wright and Bradbury Thermocouples

T_m °K.	Wright W1/3ΔT μv./°K.*	Wright W2/3ΔT μv./°K.*	Bradbury (calibration) μv./°K.
64.780	14.9	13.6	13.2
67.987	15.0	13.8	13.6
74.128	15.5	14.4	14.4
87.979	17.2	16.2	16.0
88.332	16.4	15.9	16.1
92.546	17.0	16.5	16.9
99.291	17.7	17.2	17.4
99.980	18.0	17.1	17.5
109.986	18.9	18.5	18.7
110.724	18.8	18.5	18.8
111.122	18.8	18.5	18.9
118.938	19.6	19.0	19.7
121.268	19.9	19.6	20.1
121.635	20.3	19.9	20.1
135.836	21.2	20.9	21.8
137.523	21.4	21.5	22.0
145.886	22.2	22.5	23.0

*per junction

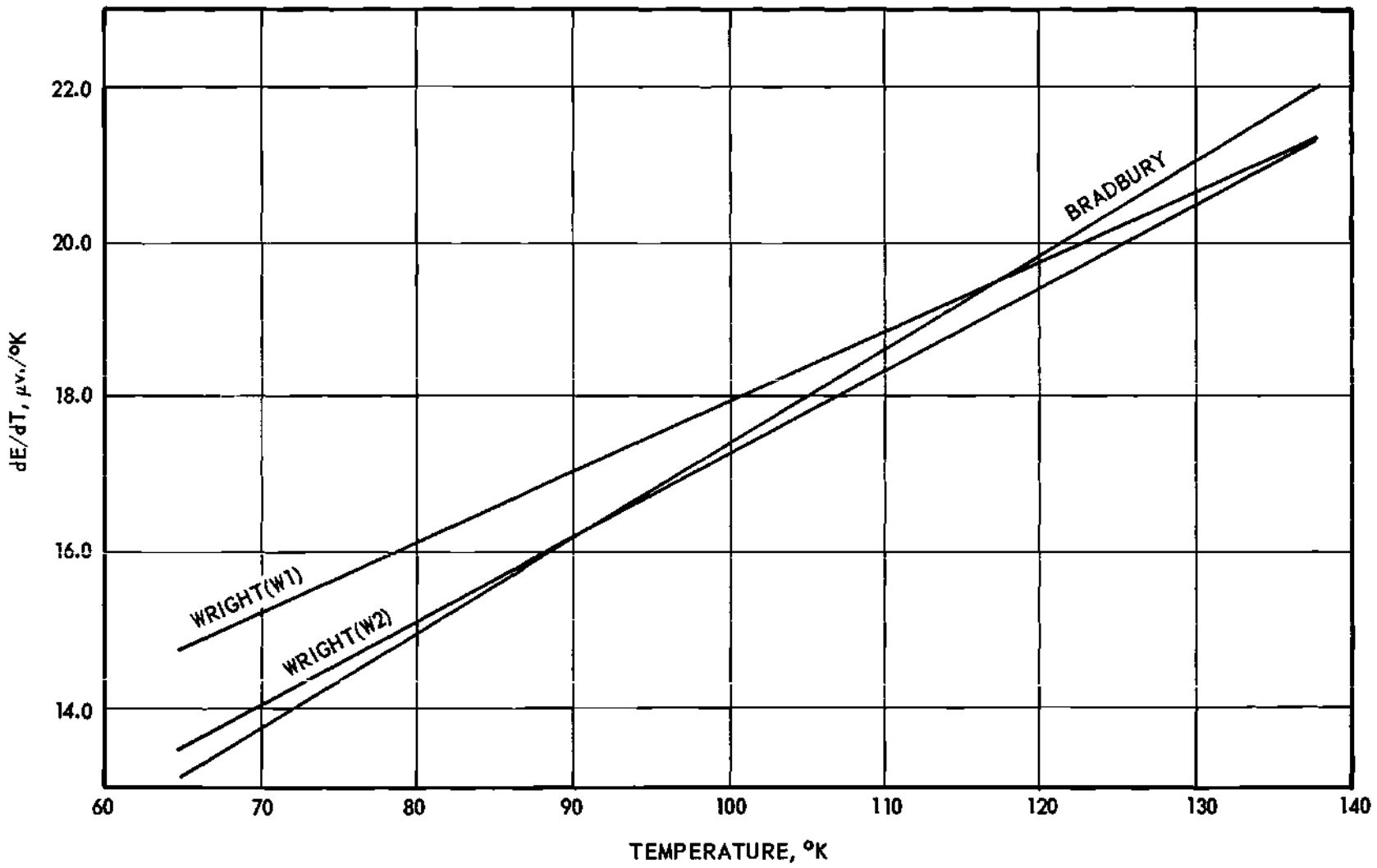

Figure 9. Variations of dE/dT with Temperature for Wright and Bradbury Thermocouples.

divergences were noted for W1. The handling of the Wright thermocouples may have introduced strains that caused a fraction of the observed differences.

Comparison of the temperature sensitivities (i.e. dE/dT) exhibited by the Wright thermocouples, W1 and W2, in the Wright and Bradbury cryostats can be made with Figures 10 and 11. In these figures, Wright calibration data (see Table 7 of Wright's thesis) for the three-junction thermocouples are plotted along with the data obtained from the same thermocouples while installed in the Bradbury cryostat. Both W1 and W2 showed consistently larger dE/dT values with the Bradbury cryostat, W1 producing the greater difference in sensitivity. For the temperature range, 80 to 130° K., the W1 data from this work was about 11 per cent greater than that recorded by Wright for the same thermocouple. The W2 thermocouple readings from the two cryostats differed, on the average, about 3 per cent over the same temperature range.

The thermal conductivity values reported by Wright, listed in Table 4 of Chapter III, had also been calculated by him using Equation (6),

$$K = \frac{L_e \, Q_e}{A \Delta T} \tag{6}$$

Since the temperature gradient, ΔT, is computed from the generated e.m.f. and the calibration of the thermocouple, i.e.,

$$\Delta T = \frac{\Delta E}{(dE/dT)} \tag{33}$$

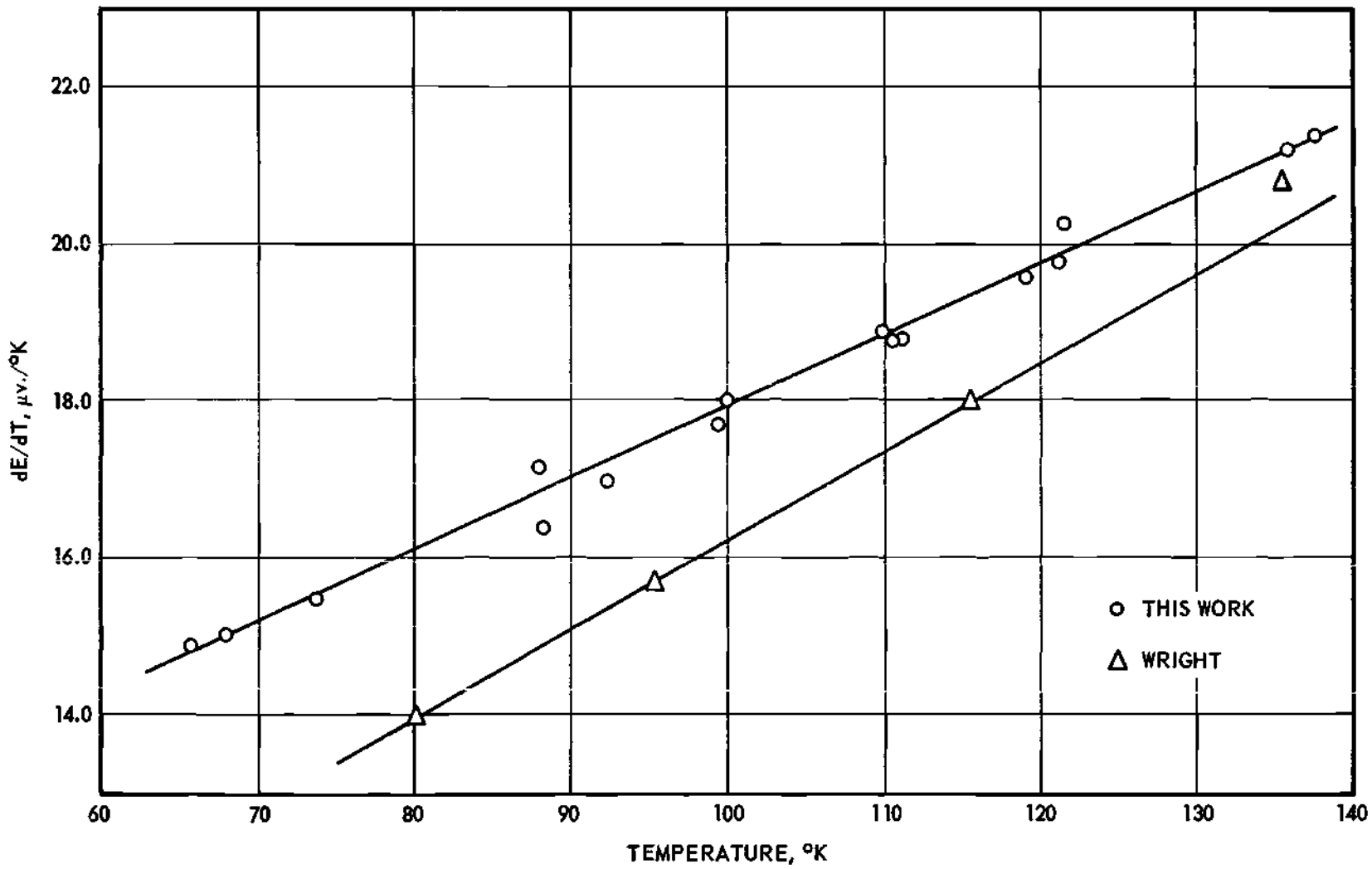

Figure 10. Variation of dE/dT with Temperature for Wright Thermocouple W1.

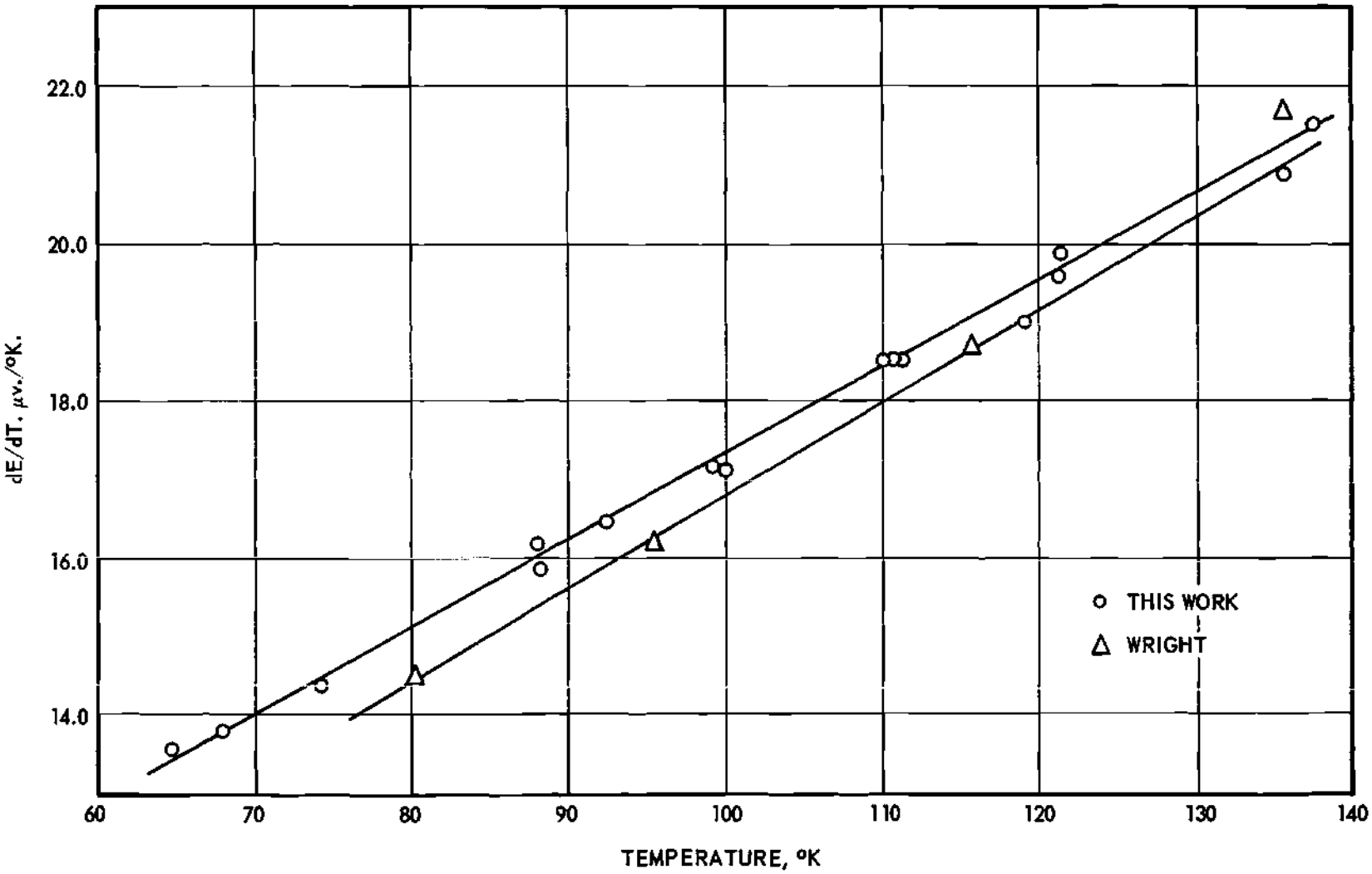

Figure 11. Variation of dE/dT with Temperature for Wright Thermocouple W2.

a thermal conductivity reflecting adjustment for the difference in thermocouple sensitivity encountered under the different operating conditions may be defined as

$$K' = \frac{L_e \; Q_e}{A \, \Delta T'} \tag{34}$$

where

$$\Delta T' = \Delta T \, \frac{(dE/dT)_{Wright}}{(dE/dT)_{This\ work}} , \tag{35}$$

or

$$K' = K \, \frac{(dE/dT)_{This\ work}}{(dE/dT)_{Wright}} \tag{36}$$

Taking, for example, the Wright K value of 0.940 for cadmium at 81.51° K., the corresponding W1 temperature sensitivities (read from the curves of Figure 10) are 14.12 and 16.24 microvolts per degree for the Wright and Bradbury cryostats, respectively. Insertion of these values into Equation (36) gives

$$K' = \frac{(0.940)(16.24)}{(14.12)}$$

$$= 1.082 \text{ watts/cm.-}^{o}\text{K.}$$

Table 5 contains the complete set of adjusted K values for the Wright measurements with yellow brass and cadmium, and Figures 6 and 7 show curves (dashed) plotted from these adjusted data.

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*Abbreviations for Journals correspond to those given in the "List of Periodicals" published by Chemical Abstracts (1961).

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